Integer Set Coalescing for Polyhedral Compilation

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October 21, 2014

Outline

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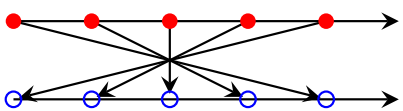
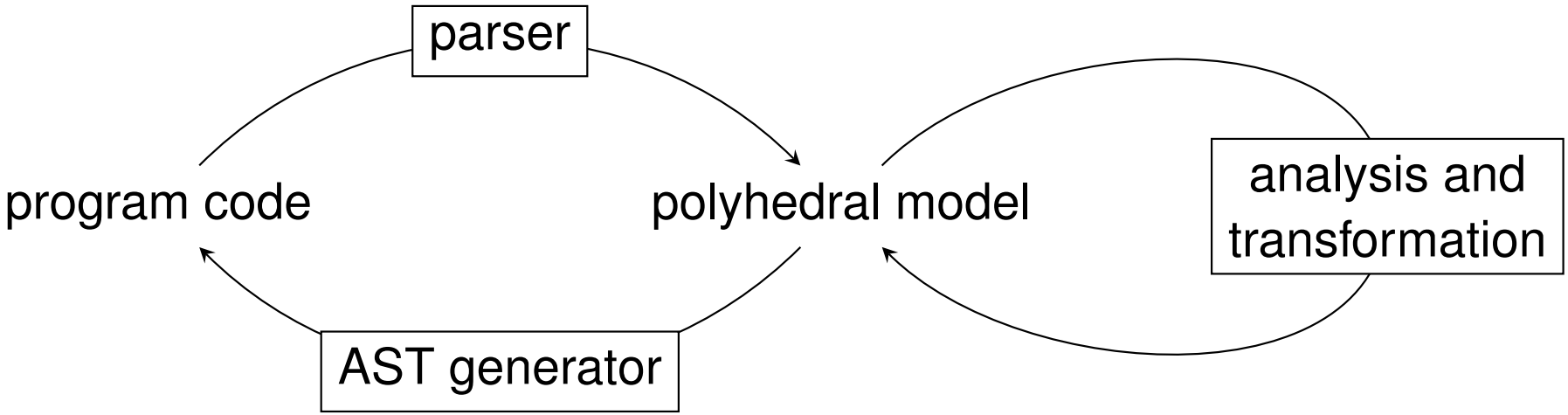
Polyhedral Compilation

```
for (i = 0; i <= N; ++i)
    a[i] = ...
for (i = 0; i <= N; ++i)
    b[i] = f(a[N-i])
```



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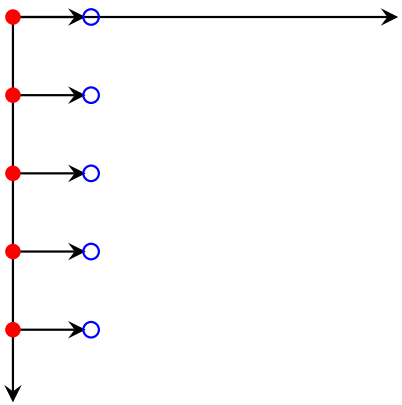

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- Instance set (set of statement instances)

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Constraints

- "read off", or
- obtained through abstract interpretation

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Equivalent Representations

extensive $\{ S(0,0); S(0,1); S(1,0); S(1,1) \}$

$= \{ S(i,j) : (i = 0 \wedge j = 0) \vee (i = 0 \wedge j = 1) \vee (i = 1 \wedge j = 0) \vee (i = 1 \wedge j = 1) \}$

intensive $\{ S(i,j) : 0 \leq i < 2 \wedge 0 \leq j < 2 \}$

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In general, representation with fewer disjuncts is preferred

- (usually) occupies less memory
- operations can be performed more efficiently
- the outcome of some operations depends on chosen representation
  - transitive closure approximation
  - AST generation

$\Rightarrow$ coalescing: replace representation by one with fewer disjuncts

Effect on AST Generation — guide

Without coalescing

```
for (int c0 = 1; c0 <= min(2 * M, N); c0 += 1)
  S1(c0);
for (int c0 = max(1, 2 * M + 1); c0 <= N; c0 += 1)
  S1(c0);
for (int c0 = N + 1; c0 <= 2 * N; c0 += 1)
  S2(c0);
```

With coalescing

```
for (int c0 = 1; c0 <= N; c0 += 1)
  S1(c0);
for (int c0 = N + 1; c0 <= 2 * N; c0 += 1)
  S2(c0);
```

Effect on AST Generation — cholesky2

Without coalescing (fragment)

```
for (int c0 = 1; c0 < 3 * M - 1; c0 += 3) {
  S3((c0 + 2) / 3);
  if (3 * M >= c0 + 8) {
    for (int c1 = (c0 + 5) / 3; c1 <= M; c1 += 1) {
      S6((c0 + 2) / 3, c1);
      for (int c4 = (c0 + 5) / 3; c4 < c1; c4 += 1)
        S5(c4, c1, (c0 + 2) / 3);
    }
  } else if (c0 + 5 == 3 * M)
    S6(M - 1, M);
}
```

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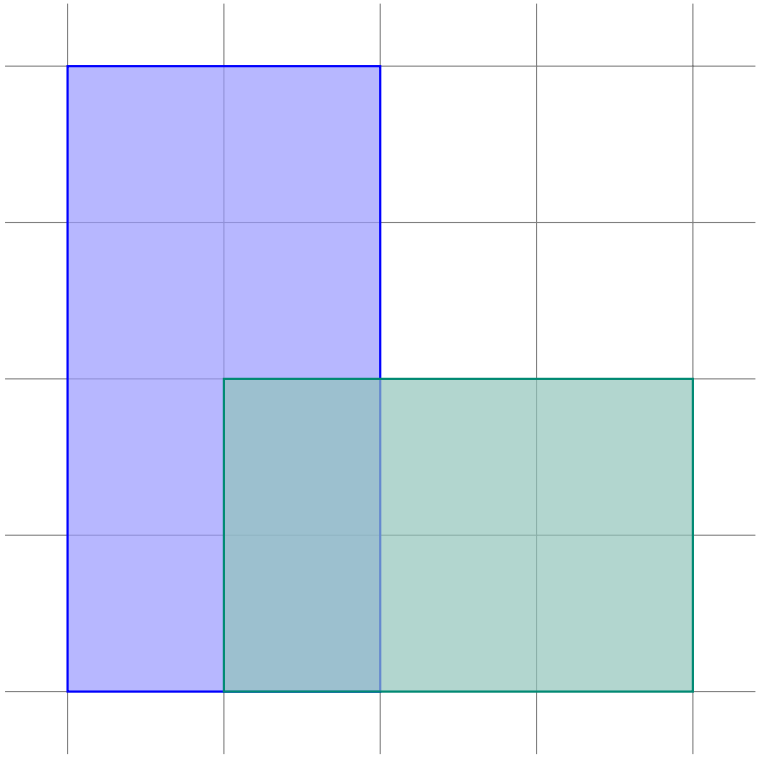
Causes of Splintering

Several operations on integer sets may introduce coalescing opportunities

- Projection



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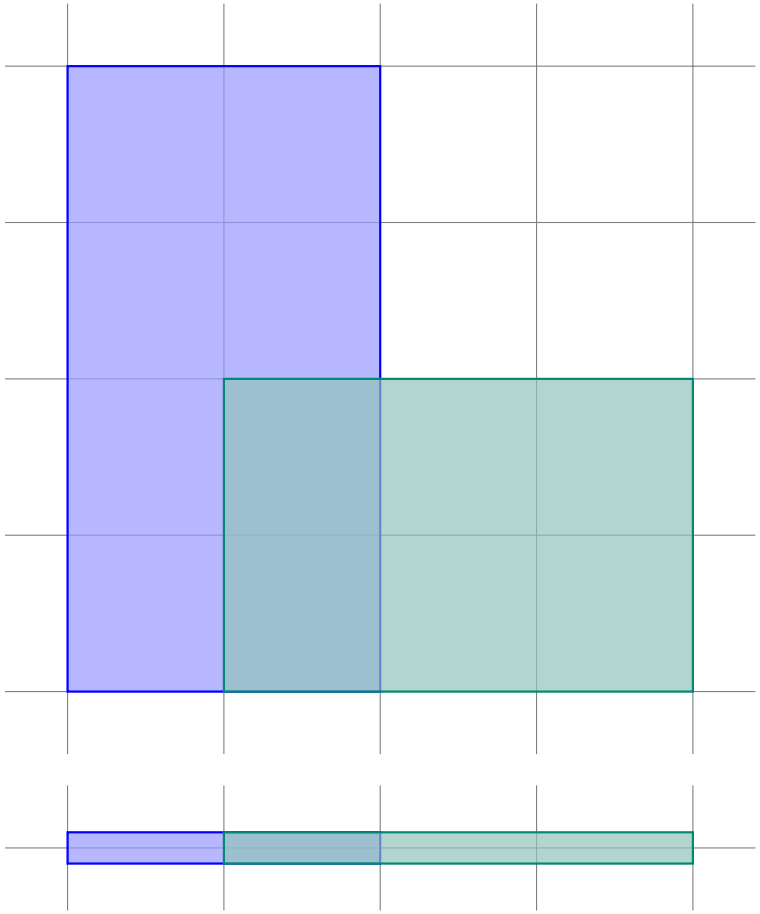

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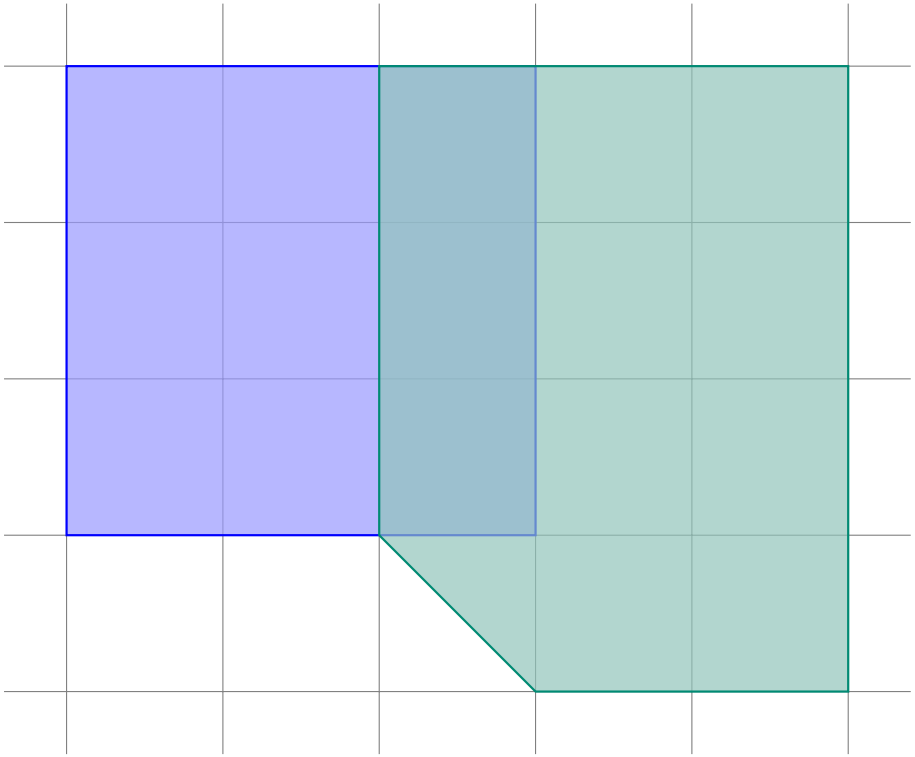

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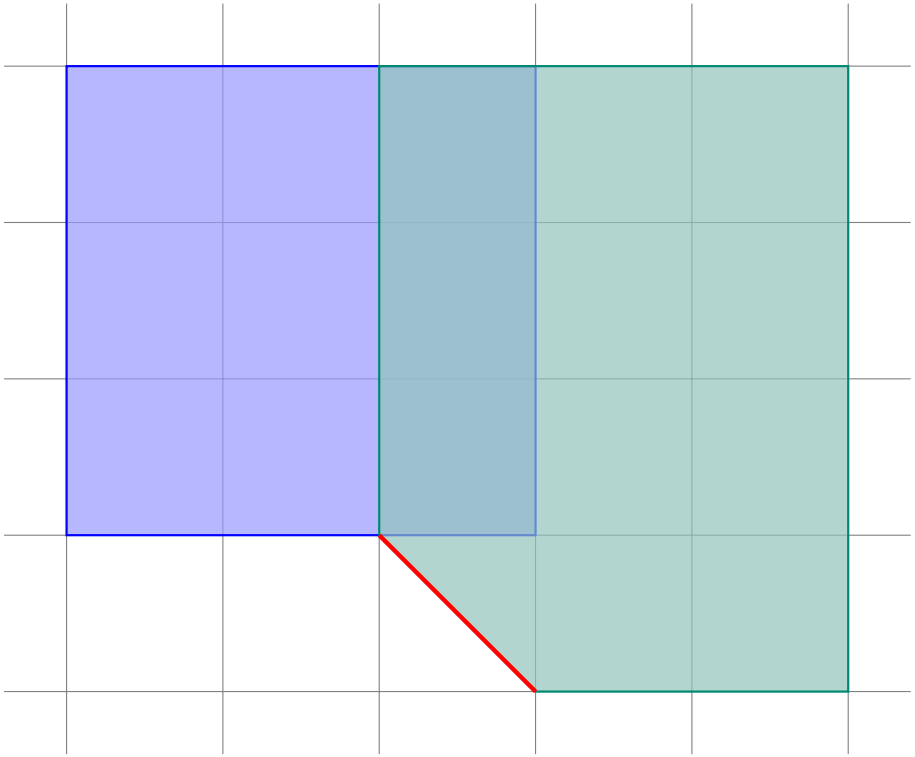

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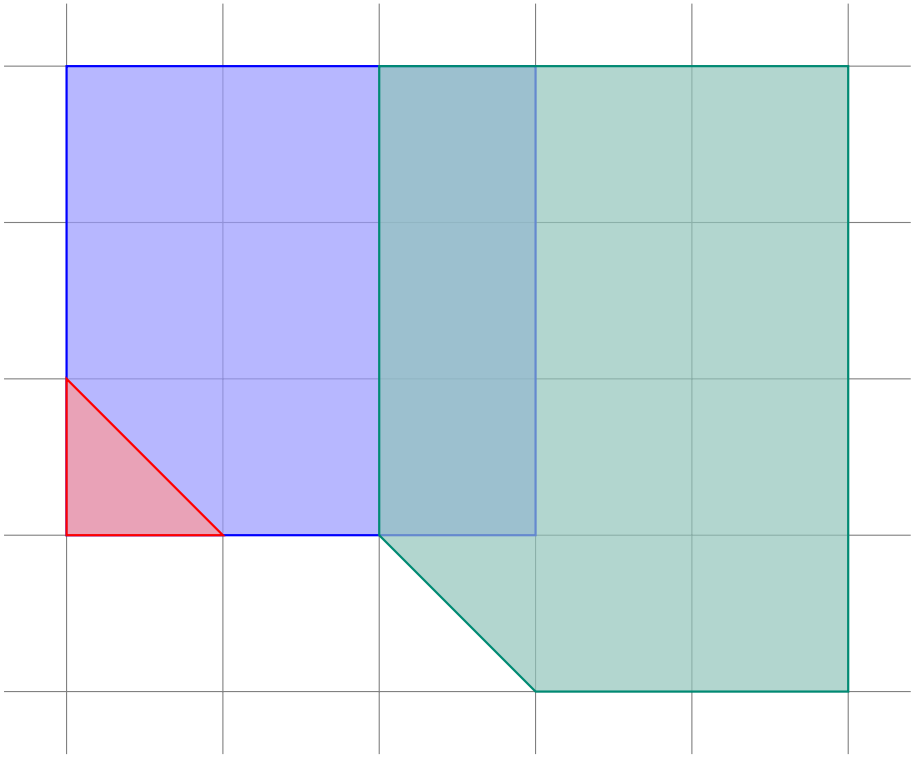

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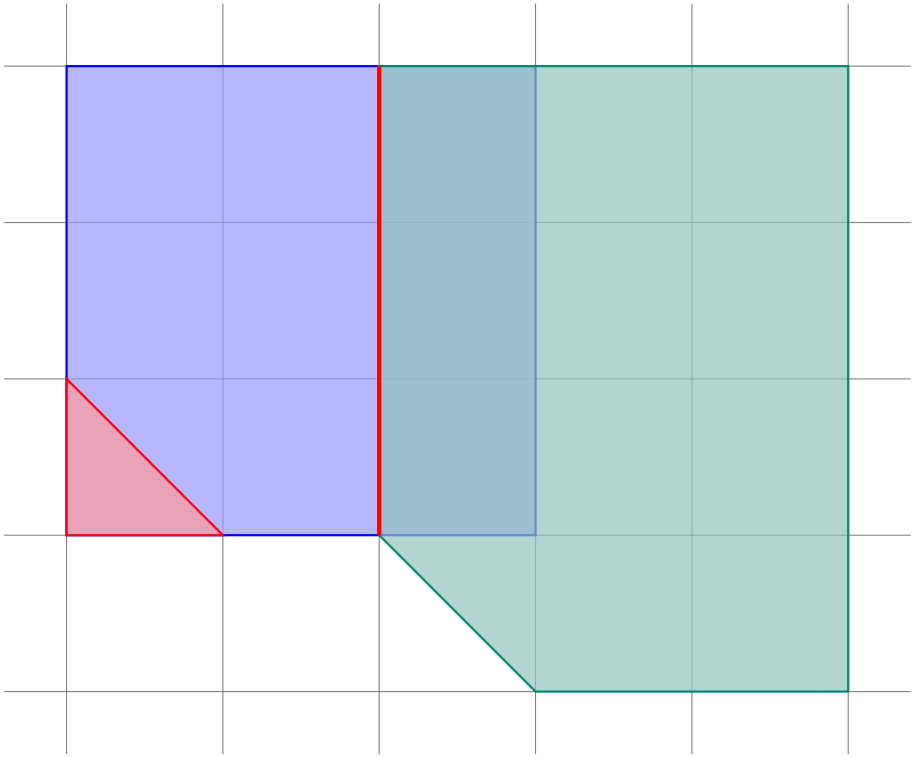

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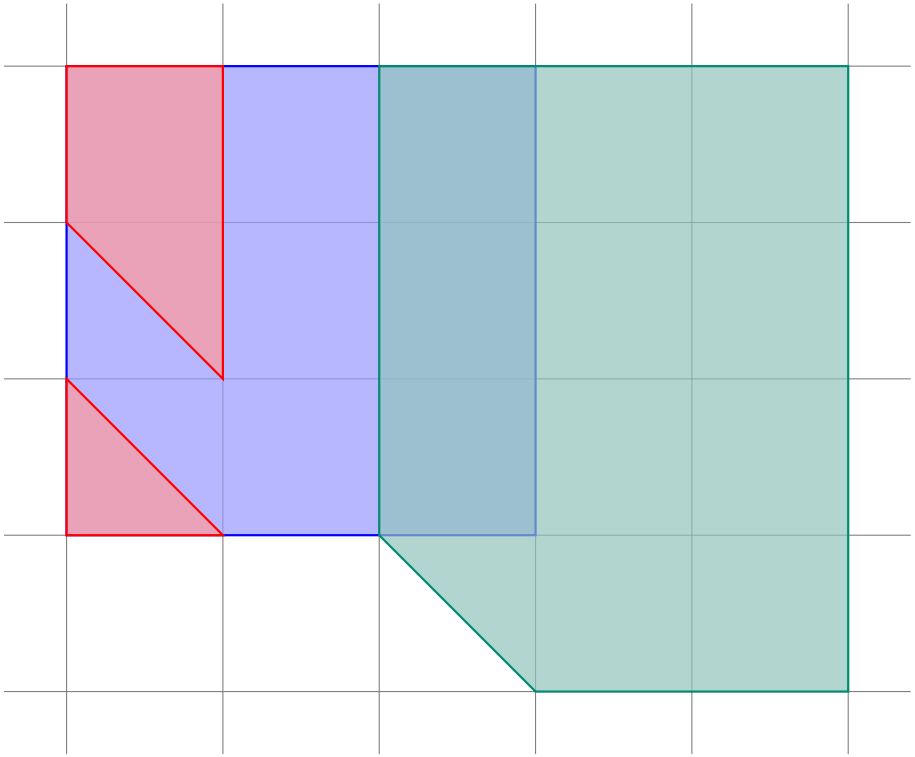

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- Projection
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- Parametric integer programming

$$\min\{(x, y) \to (z) : z \geq 0 \wedge x + z \geq 0 \wedge x + y \geq 0\}$$

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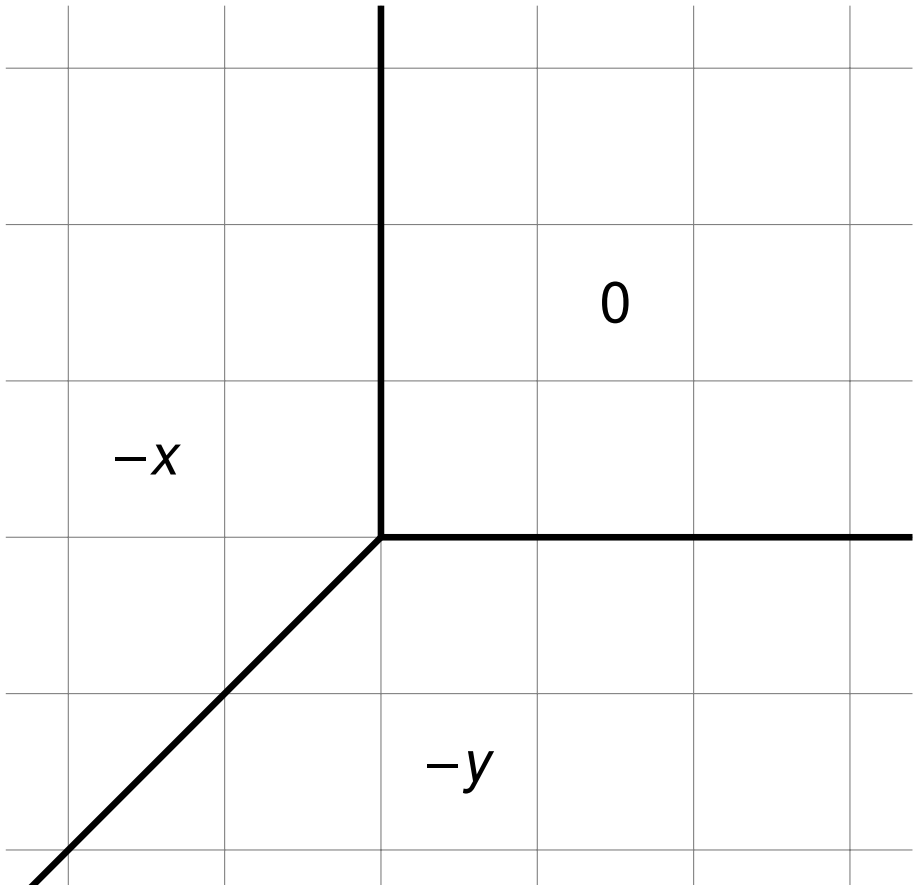

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Several operations on integer sets may introduce coalescing opportunities

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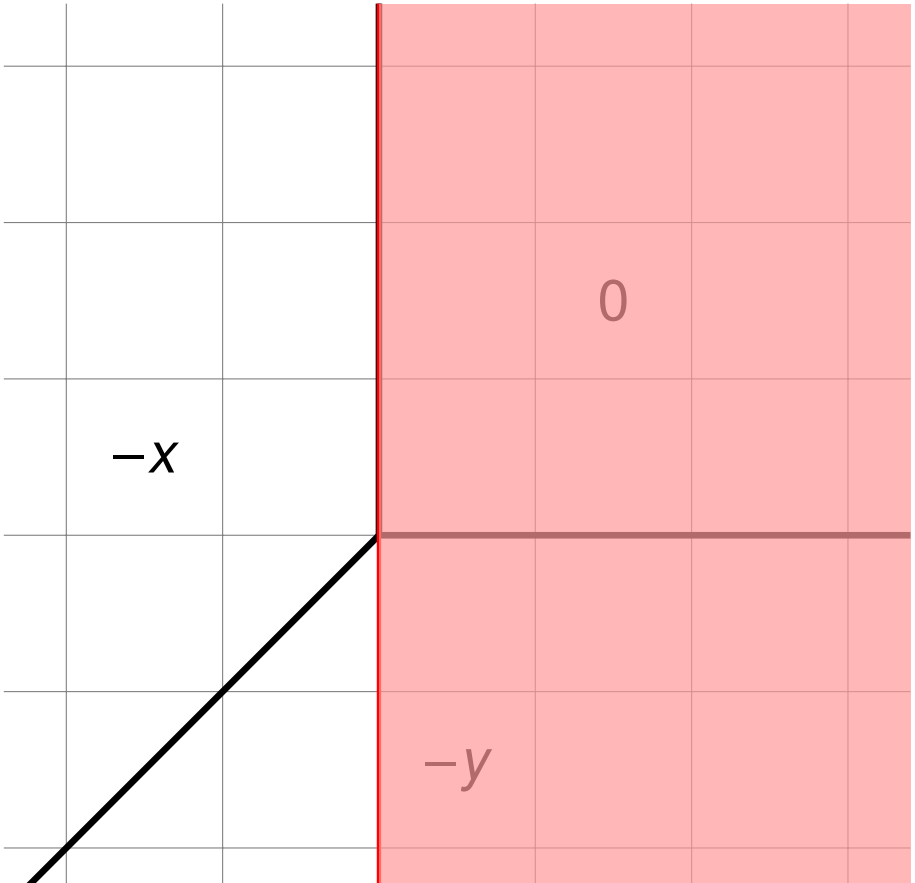

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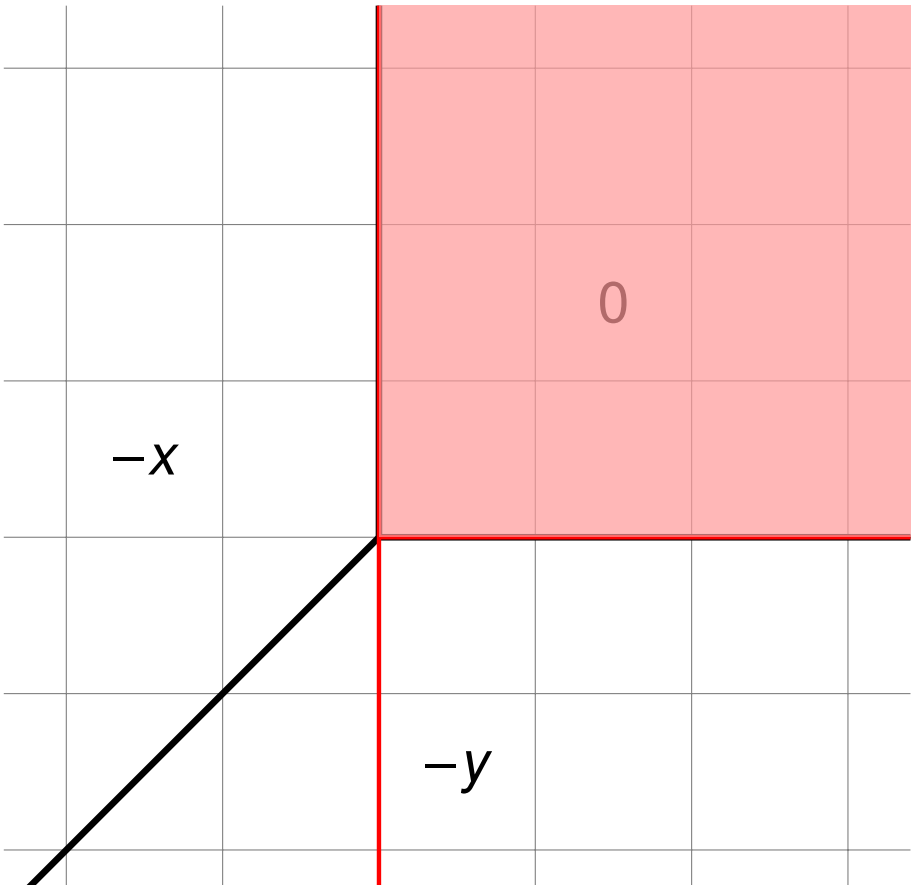

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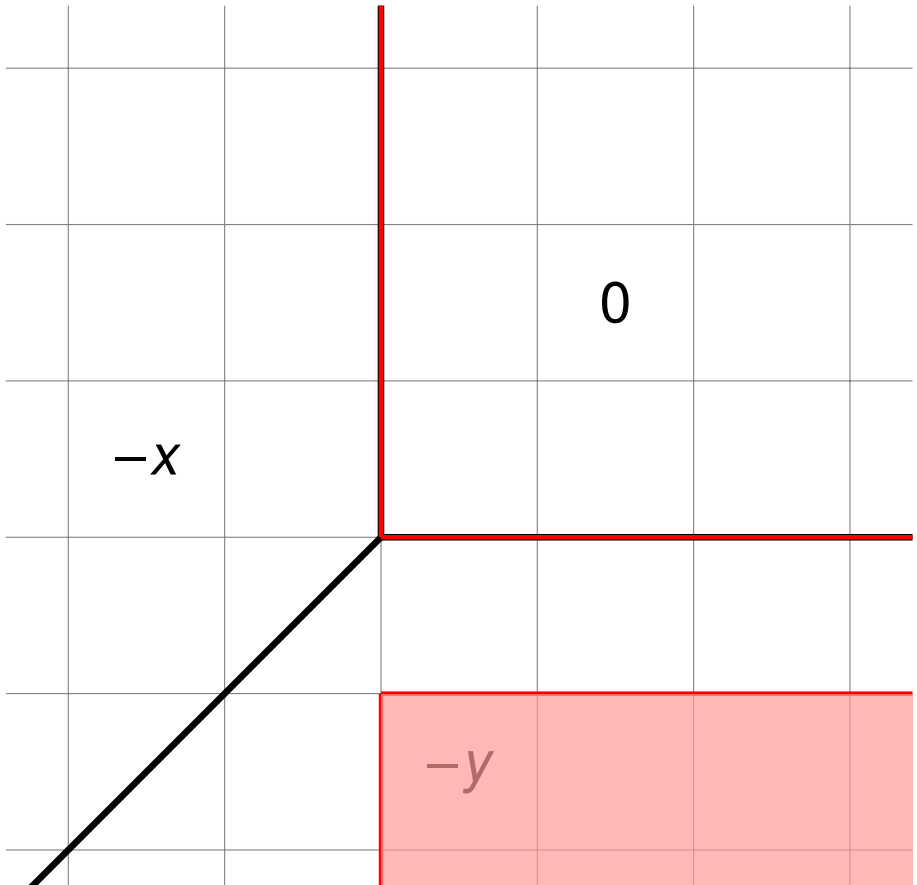

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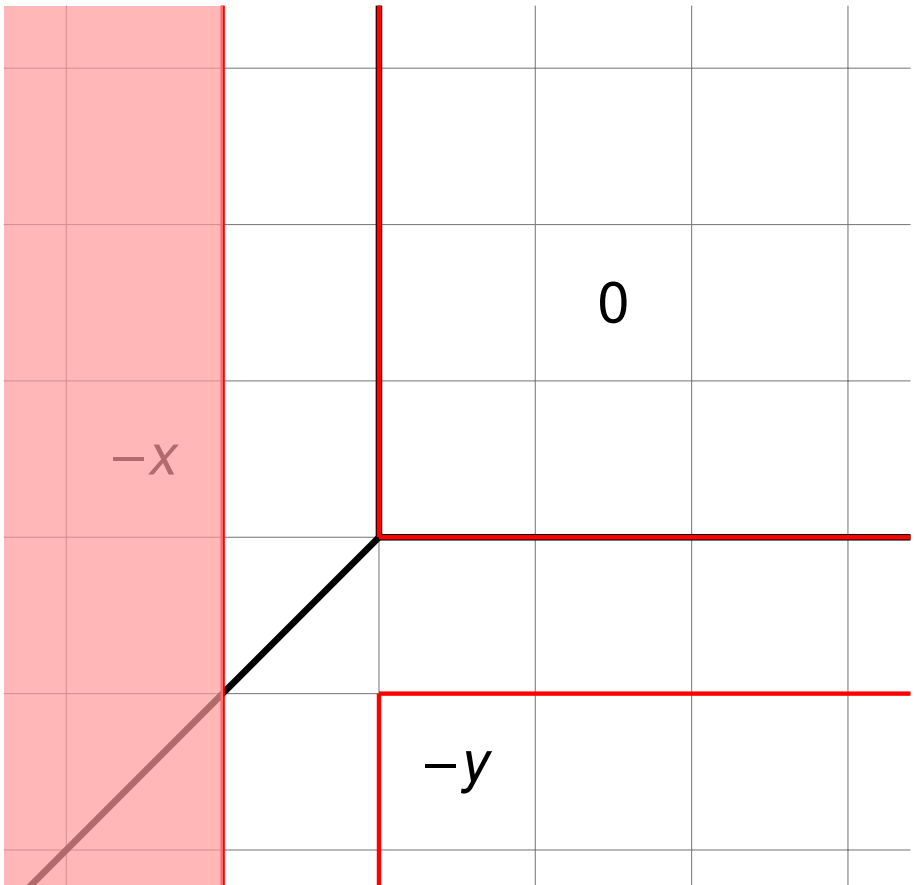

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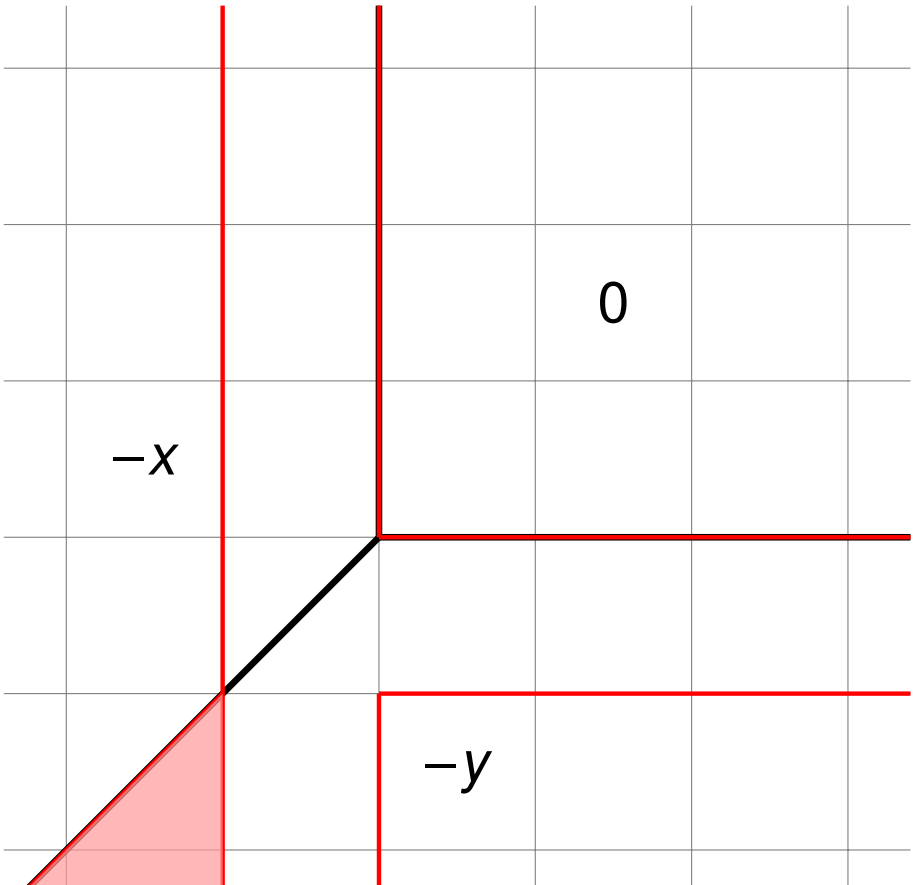

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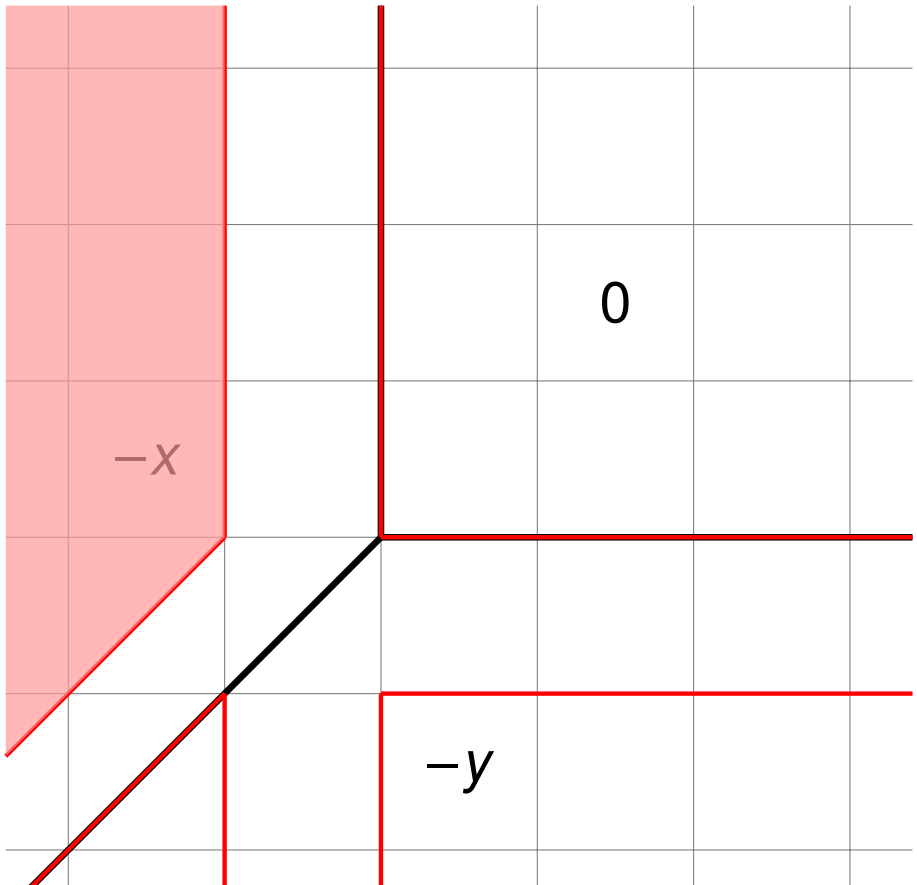

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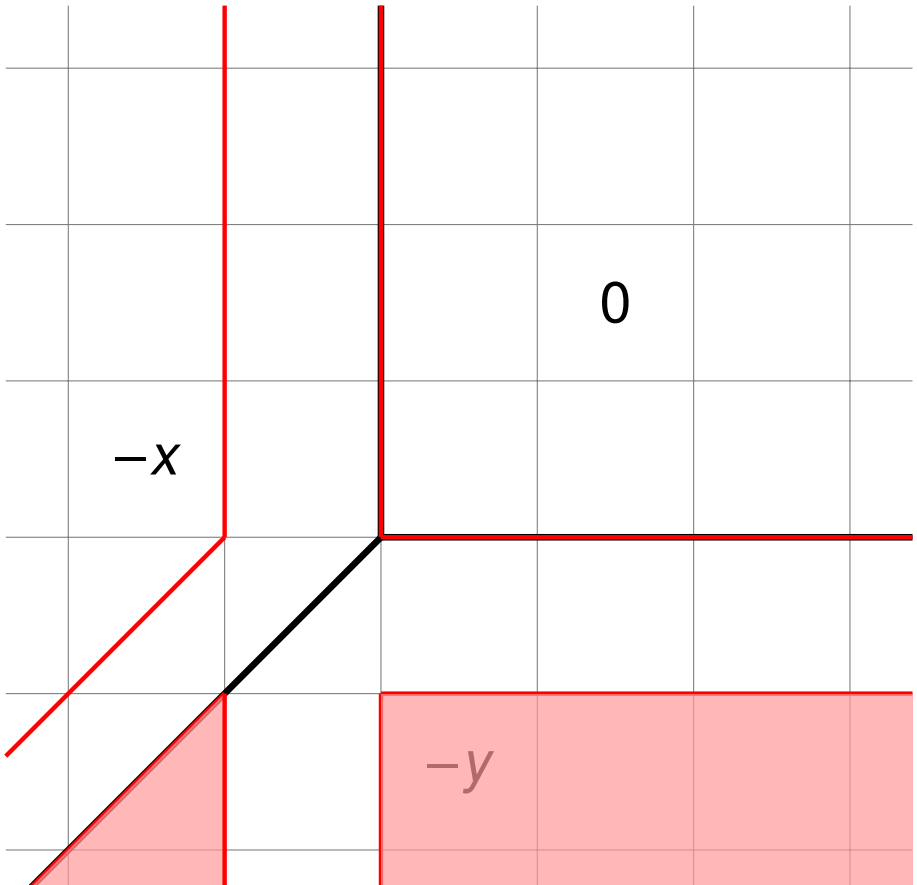

Causes of Splintering

Several operations on integer sets may introduce coalescing opportunities

- Projection
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- Union

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Traditional method (e.g., in CLooG with original PolyLib backend)

1. Compute convex hull H of S
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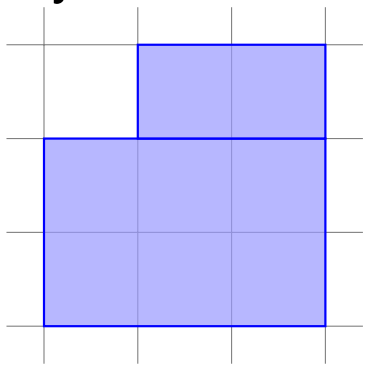

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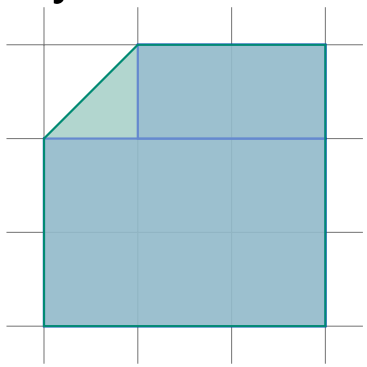

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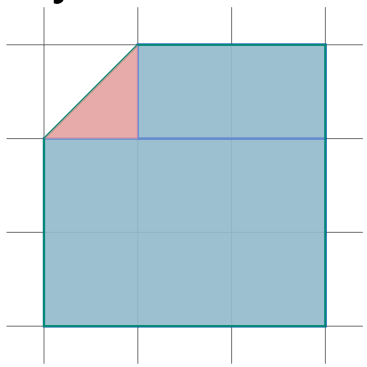

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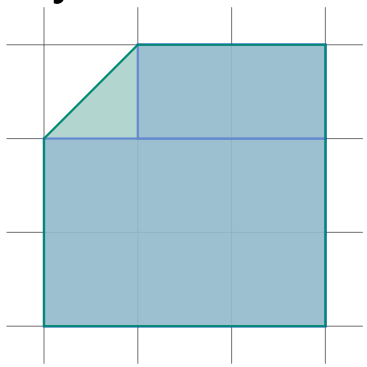

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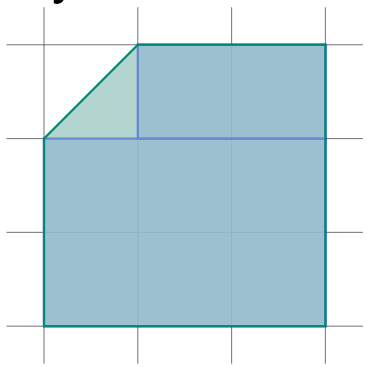

Issues:

- Convex hull may have exponential number of constraints
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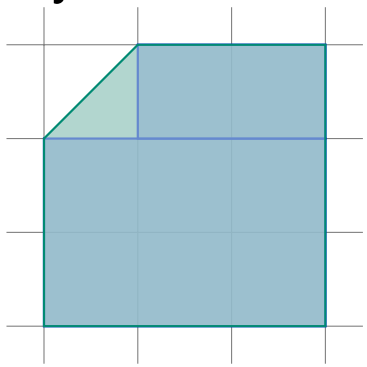

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Effect on AST Generation — covariance

With isl coalescing (in this case same result as no coalescing)

```
for (long c1 = n >= 1 ? ((n - 1) % 32) - n - 31 : 0;
     c1 <= (n >= 1 ? n - 1 : 0); c1 += 32) {
  /* .. */
}
```

With convex hull based "coalescing"

```
for (long c1 = 32 * floord(-1073741839 * n -
    32749125633, 68719476720) - 1073741792; c1 <=
    floord(715827882 * n + 357913941, 1431655765) +
    1073741823; c1 += 32) {
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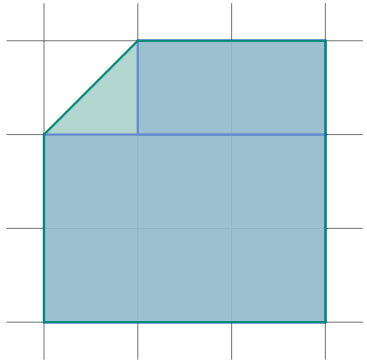

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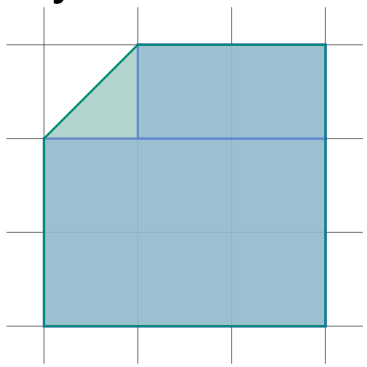

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AST Generation Times

Generation times on `isl` AST generation test cases

`isl` coalescing	16.0s
no coalescing	16.3s
convex hull (FM)	24m00s
convex hull (wrapping)	6m40s

Note: `isl` may not have the most efficient convex hull implementation

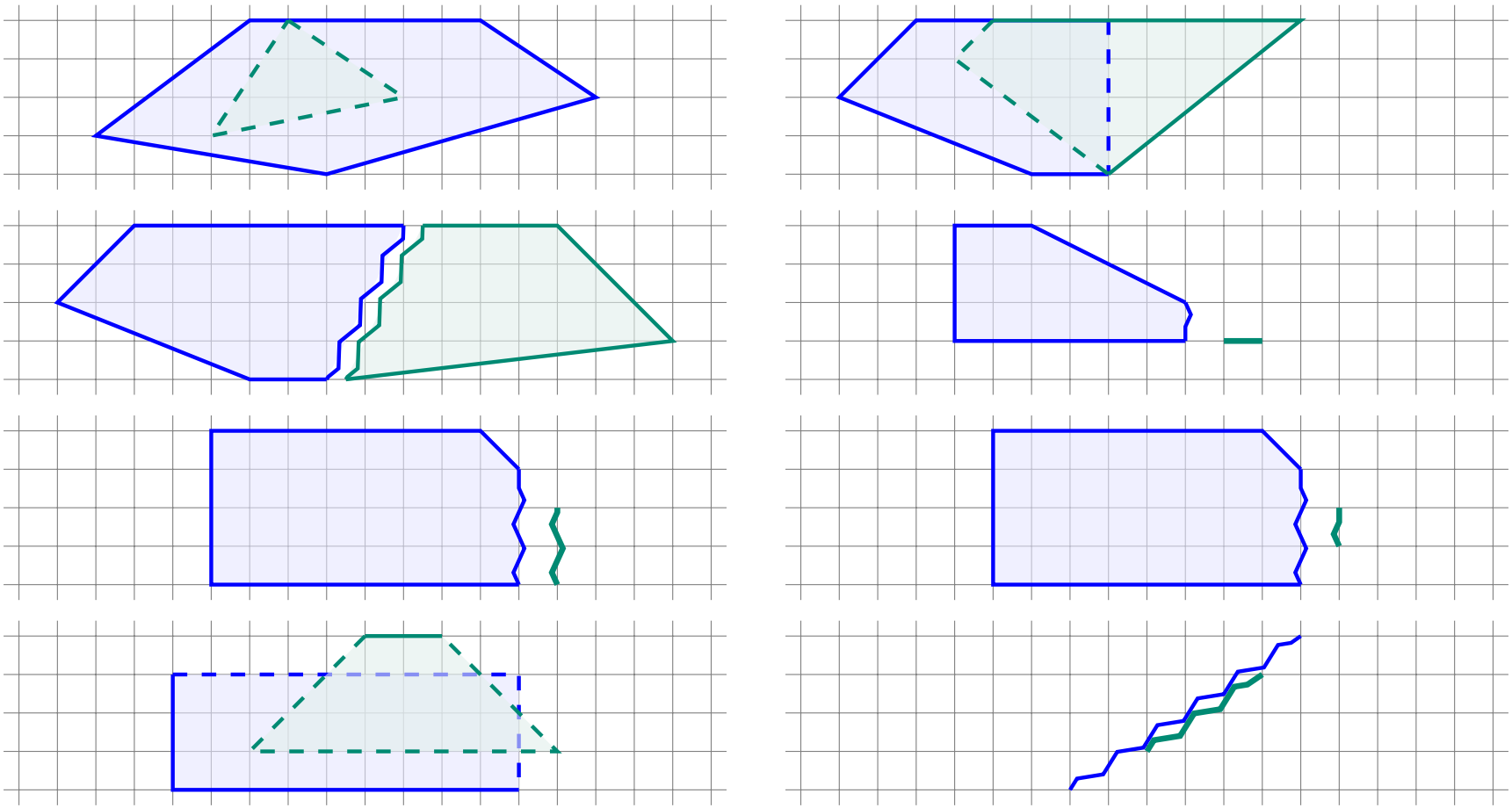
However, double description based implementations are costly too

Outline

Coalescing in isl

Coalescing in isl

- never increases the total number of constraints
- based on solving LP problems with same dimension as input set
- recognizes a set of patterns



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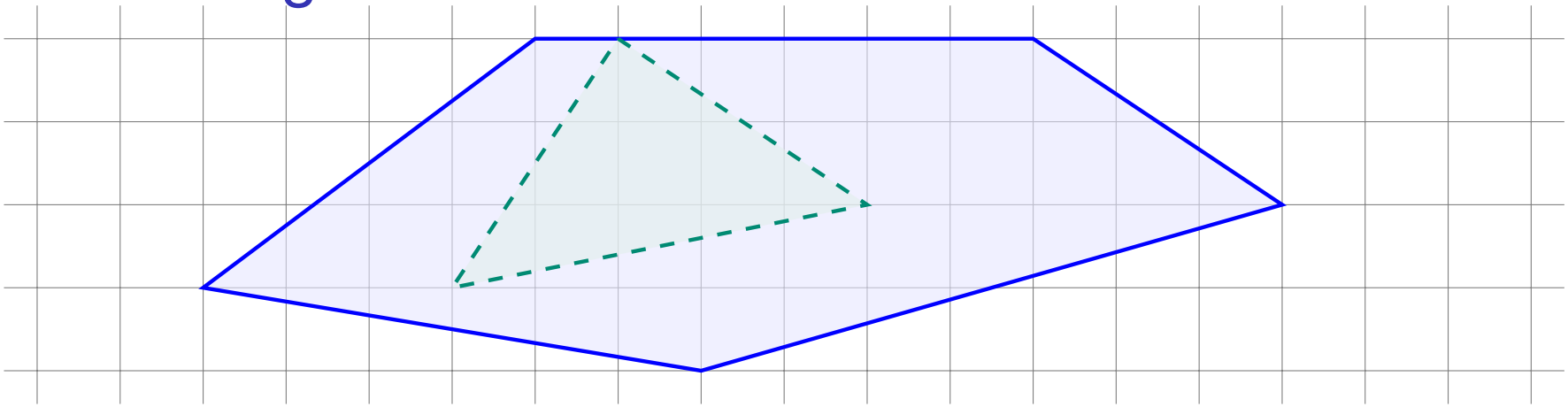
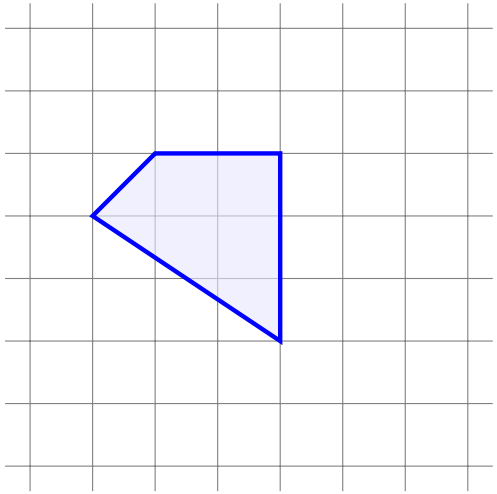


Constraint types

Given two disjuncts A and B

For each affine constraint $t(\mathbf{x}) \geq 0$ of A, determine its effect on B



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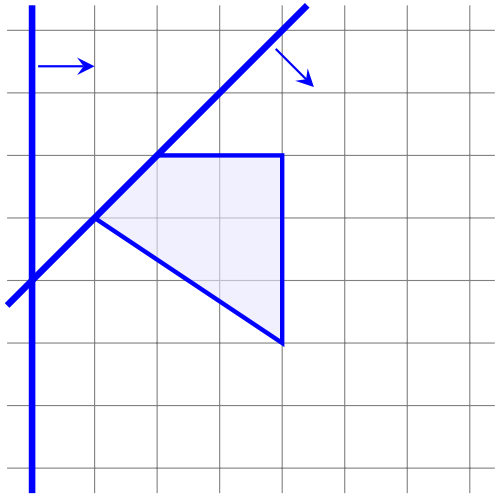

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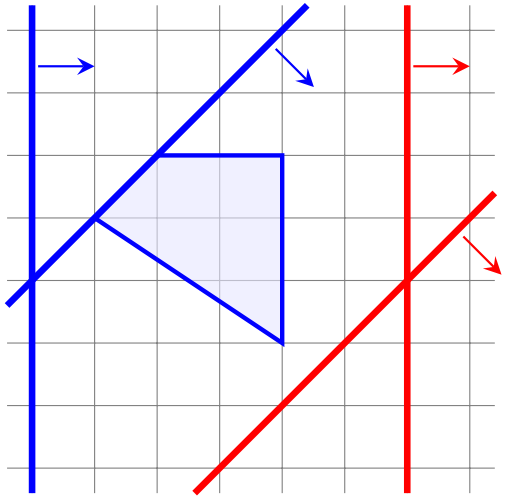

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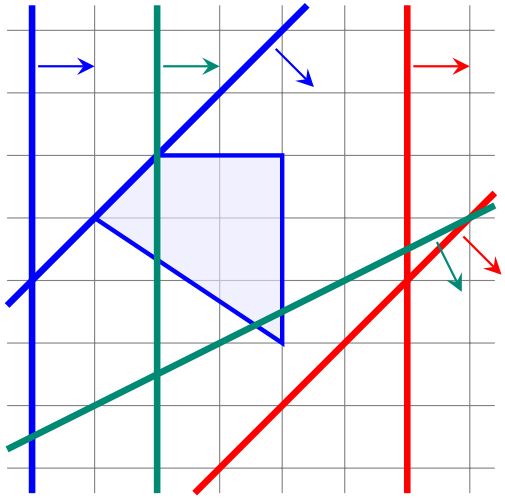

- otherwise (attains both positive and negative values over B)
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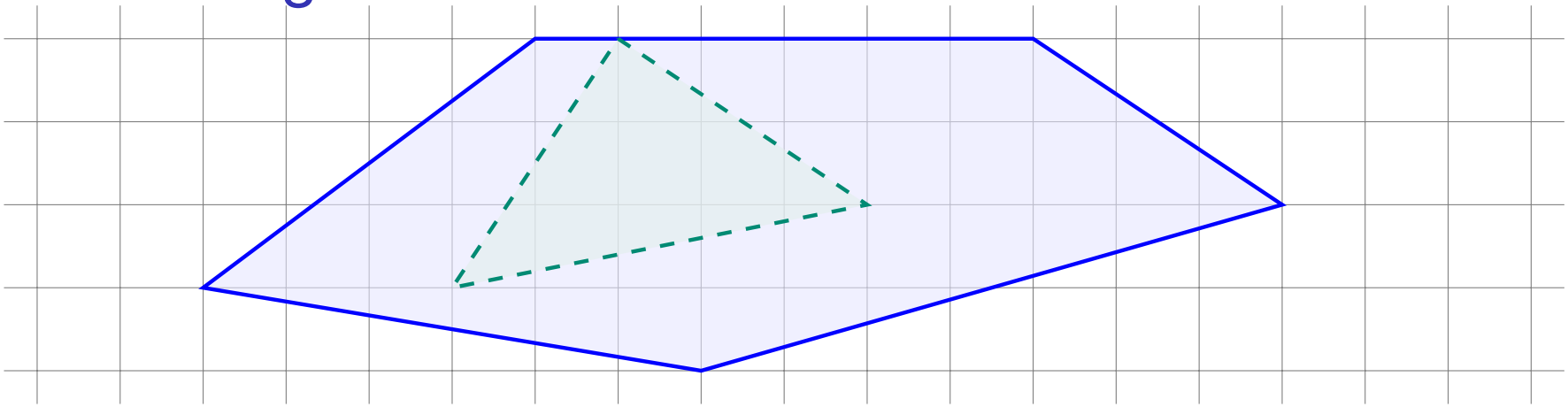

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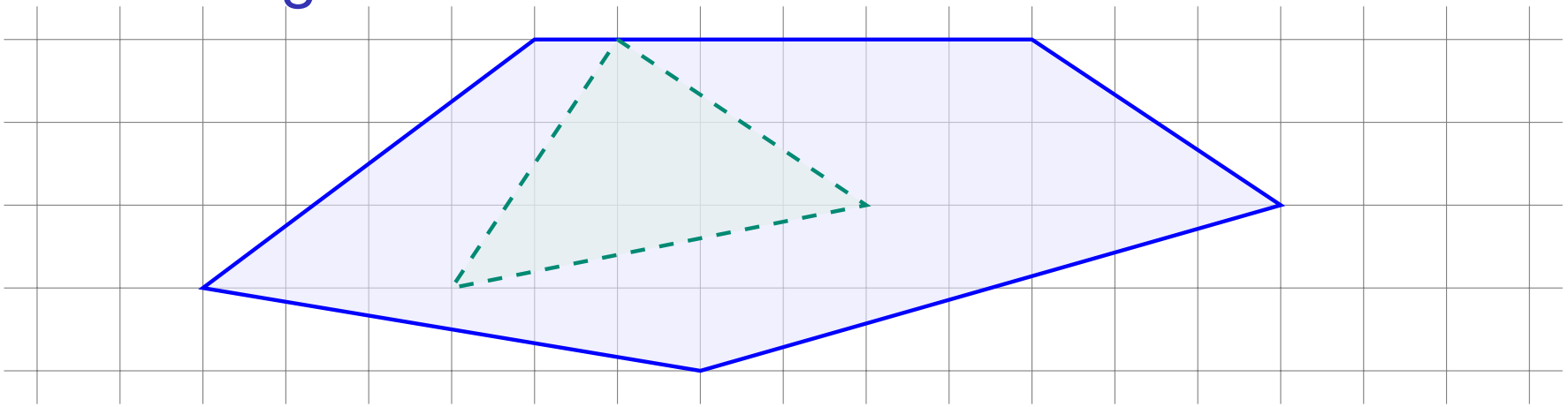

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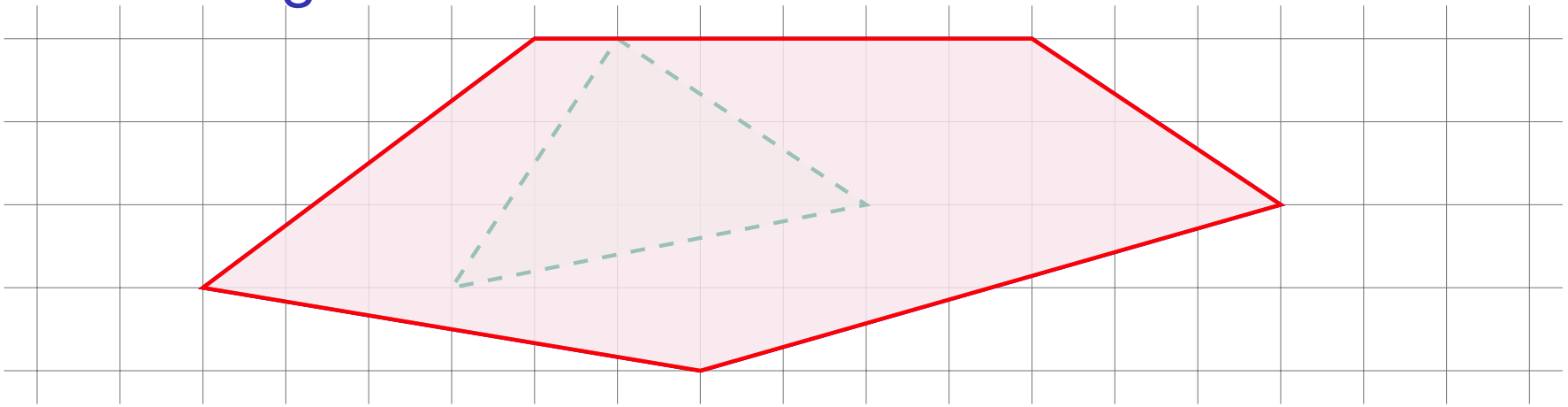

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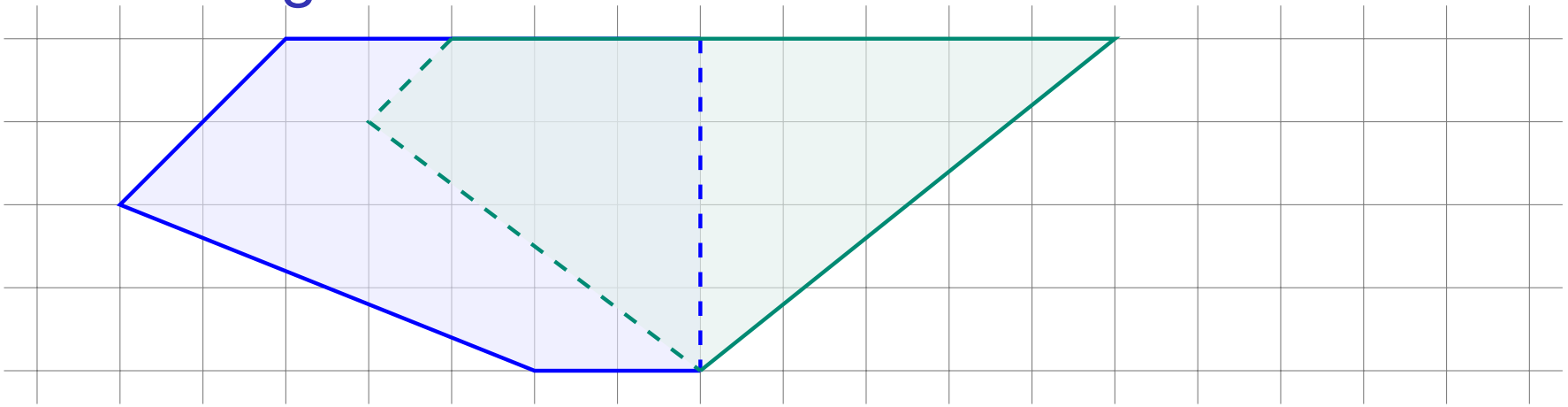

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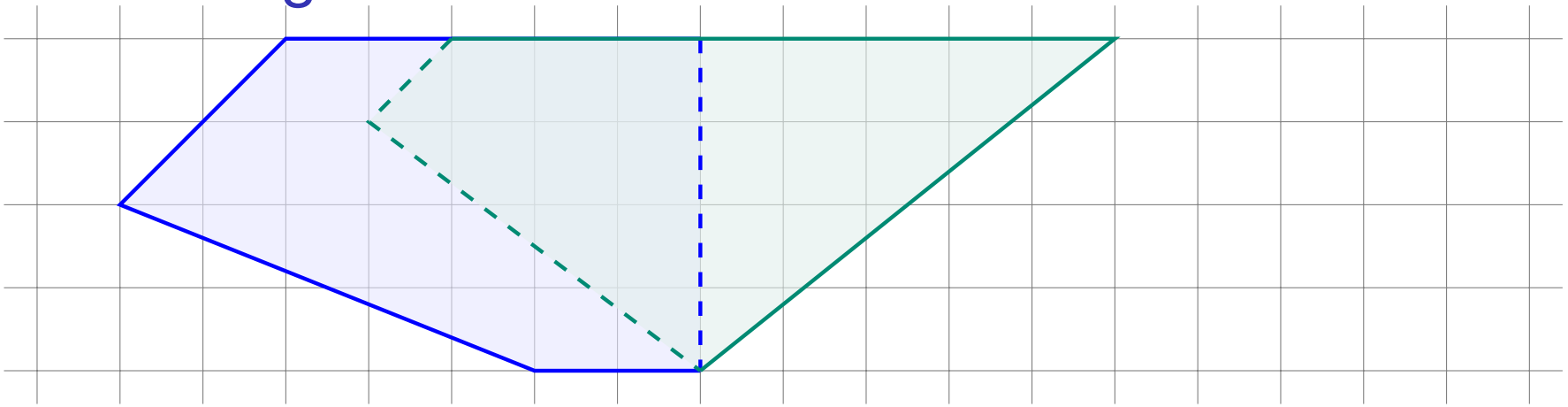

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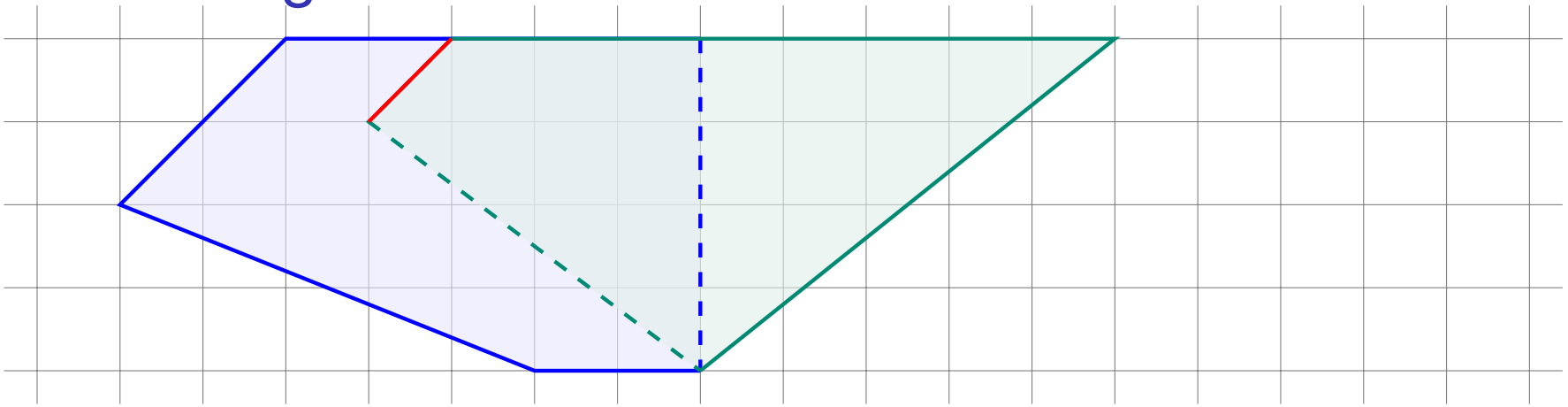

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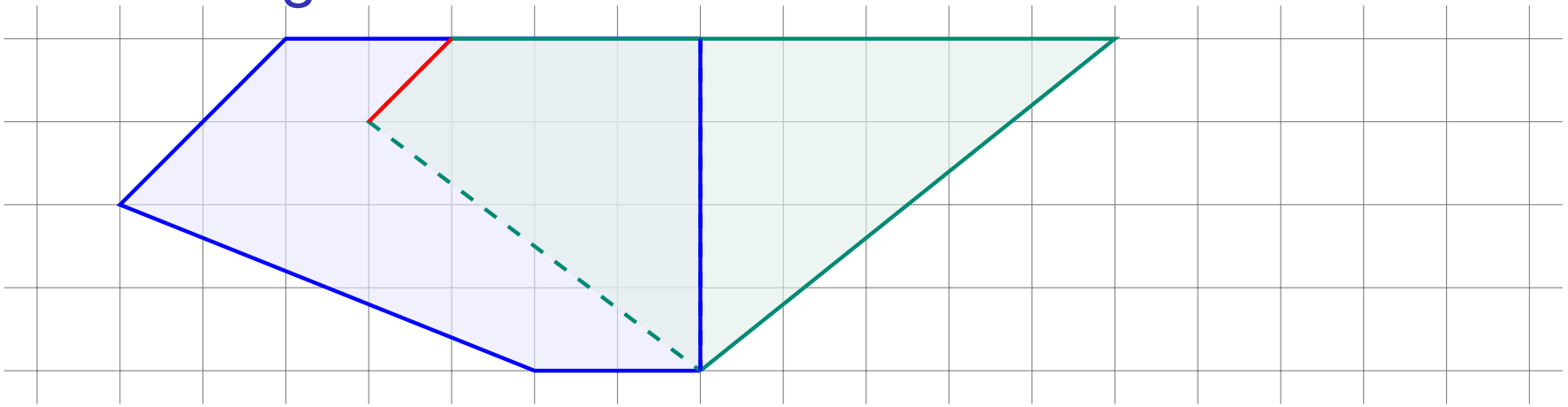

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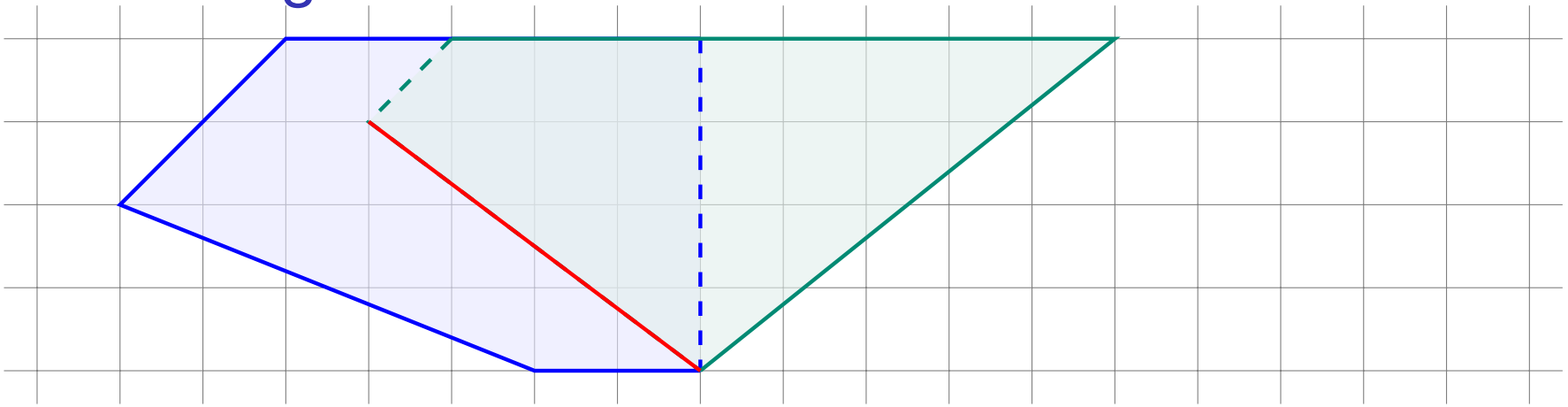

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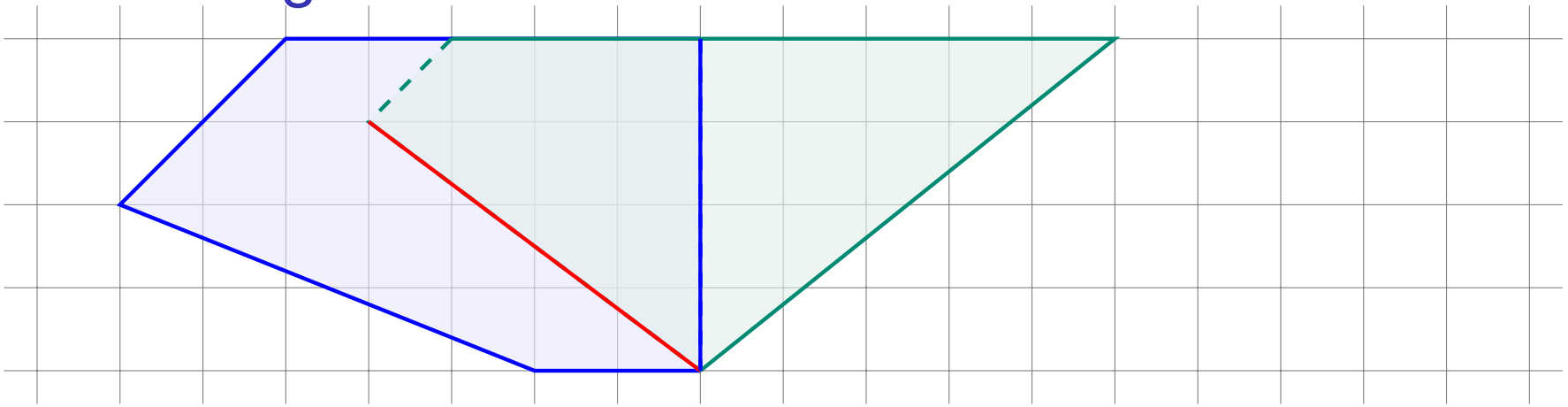

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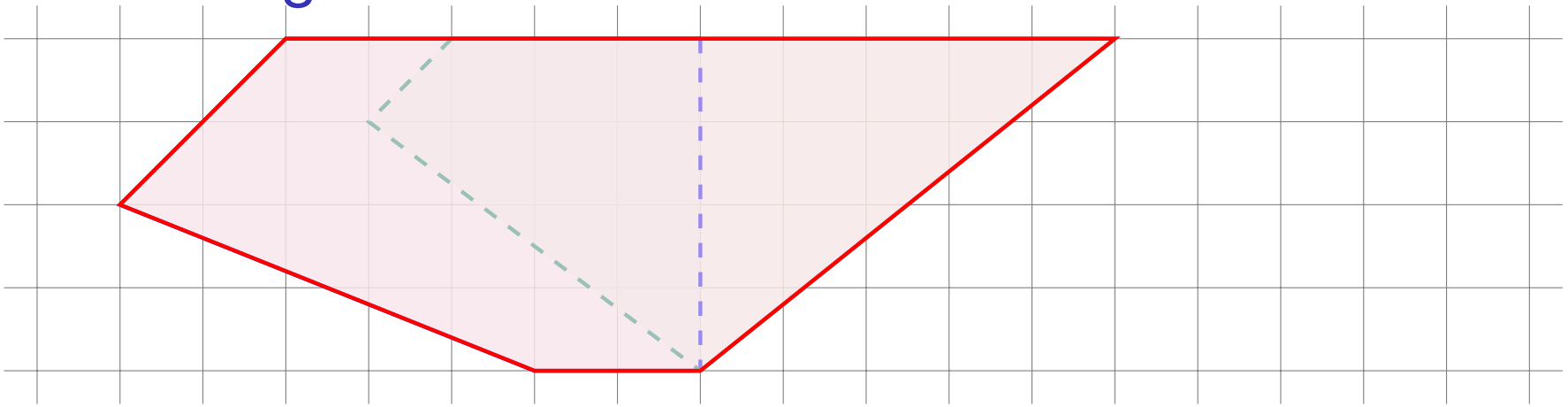

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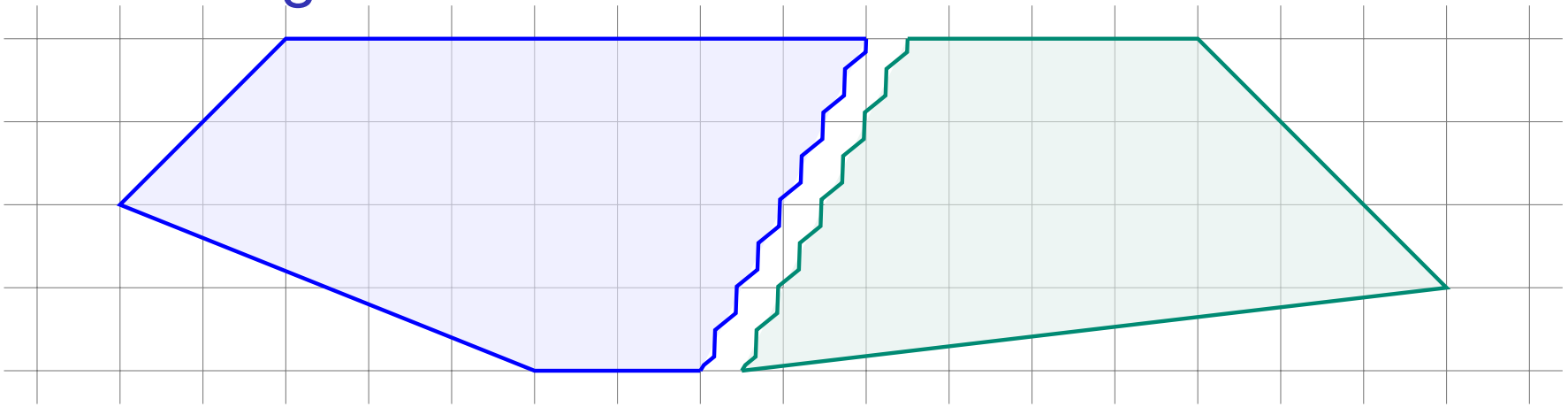

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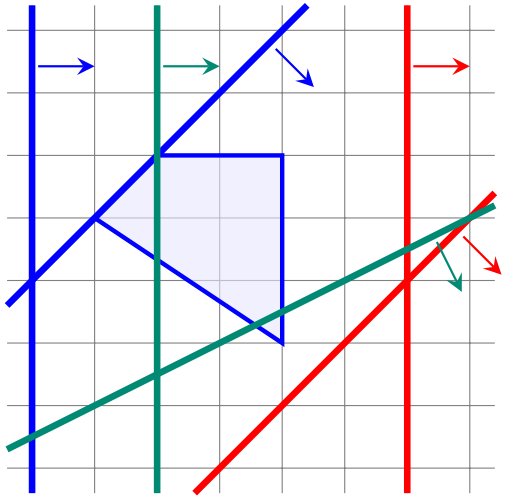

- otherwise (attains both positive and negative values over B)
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Note: affine expression $t(\mathbf{x}) \geq 0$ has integer coefficients
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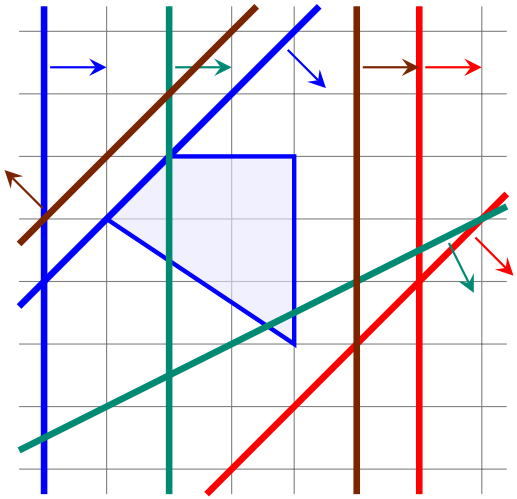

- $\min t(\mathbf{x}) > -1$ over B
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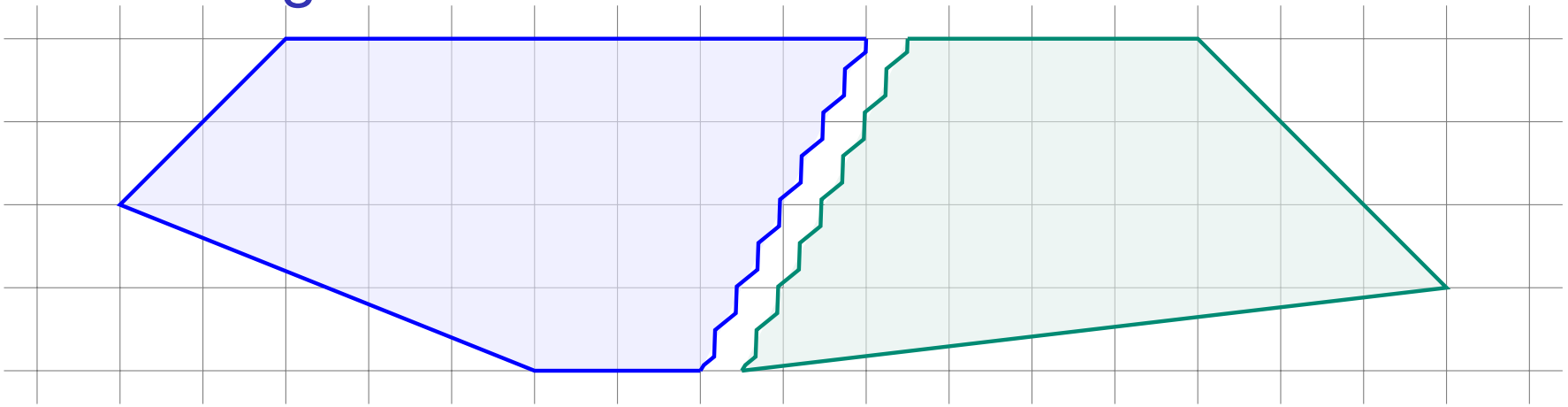
Coalescing Cases



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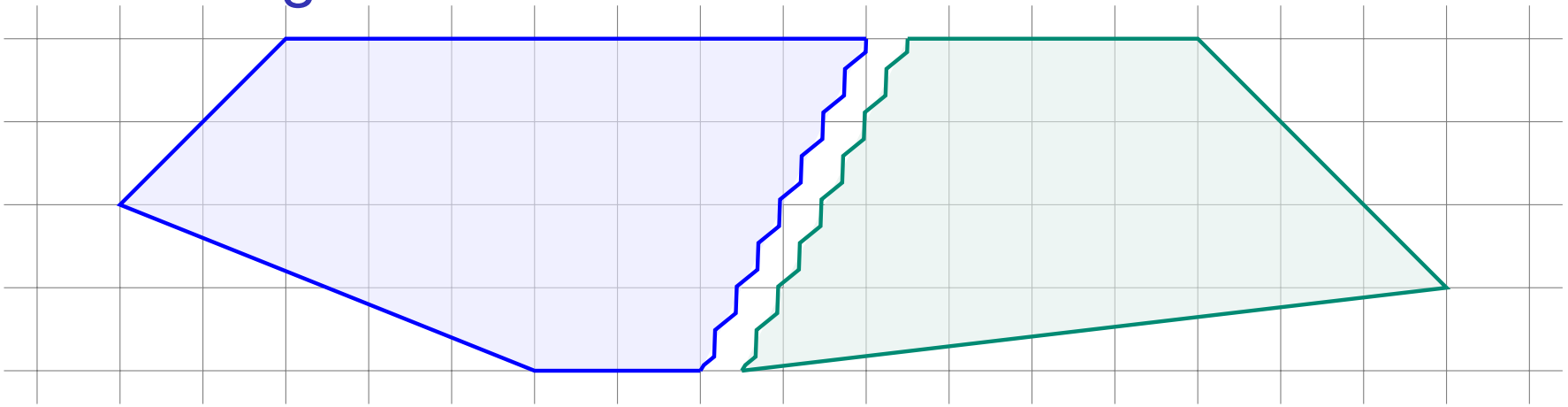

3. single pair of adjacent inequalities (other constraints valid)

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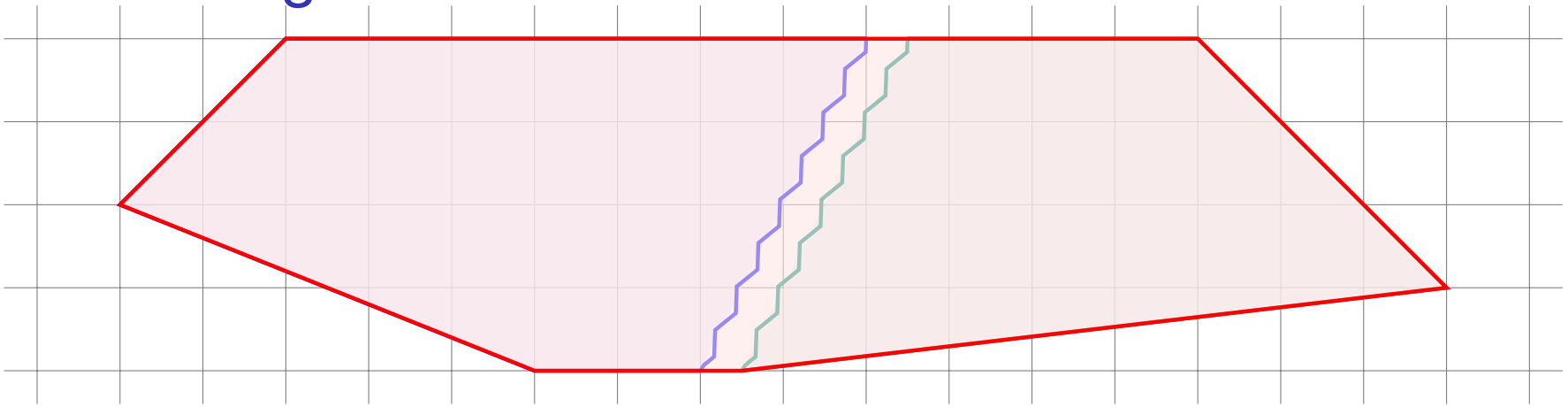

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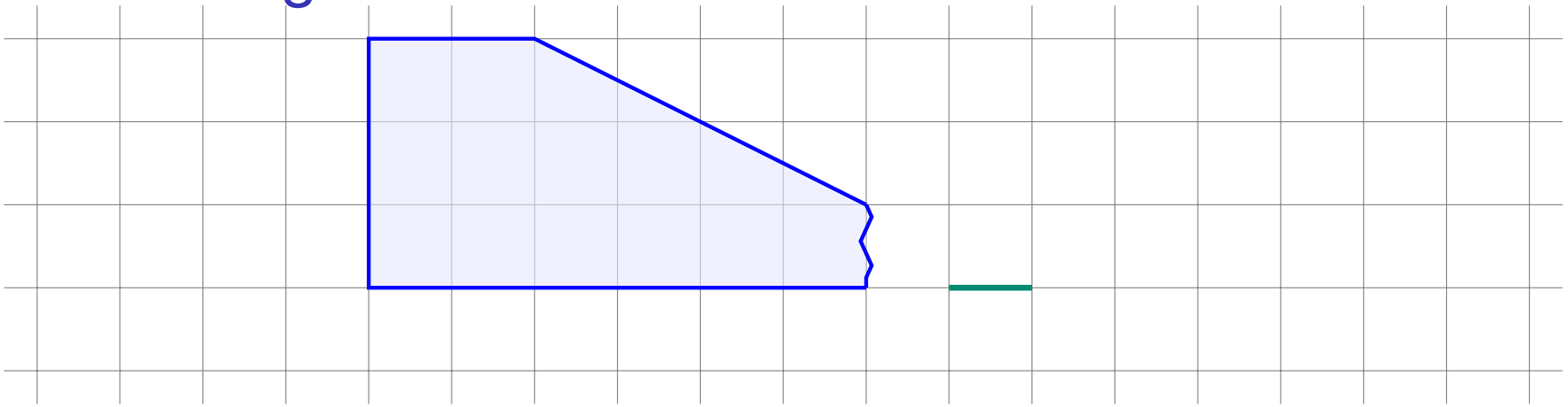

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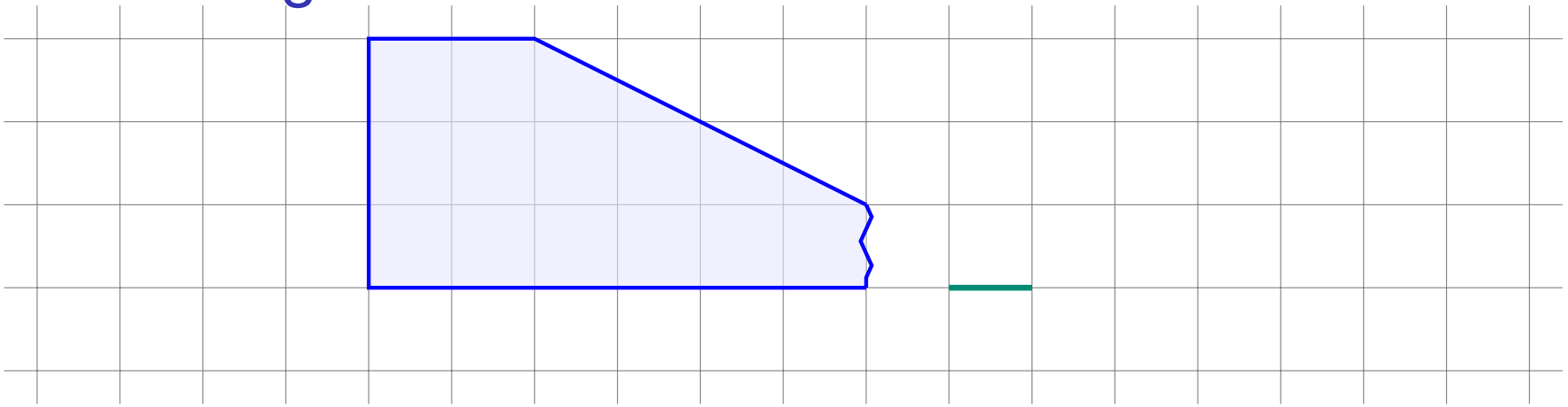

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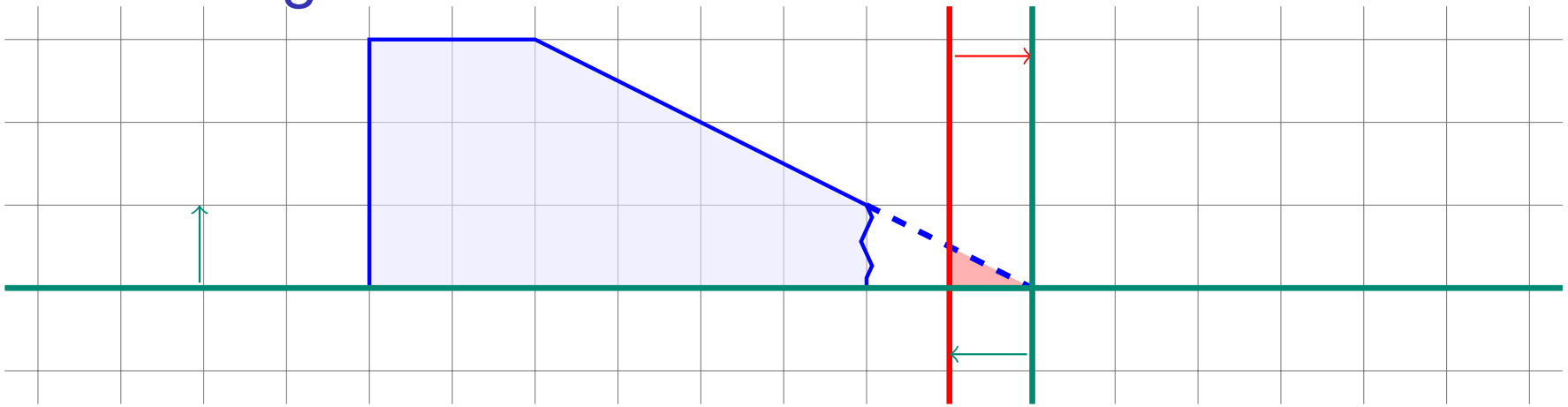

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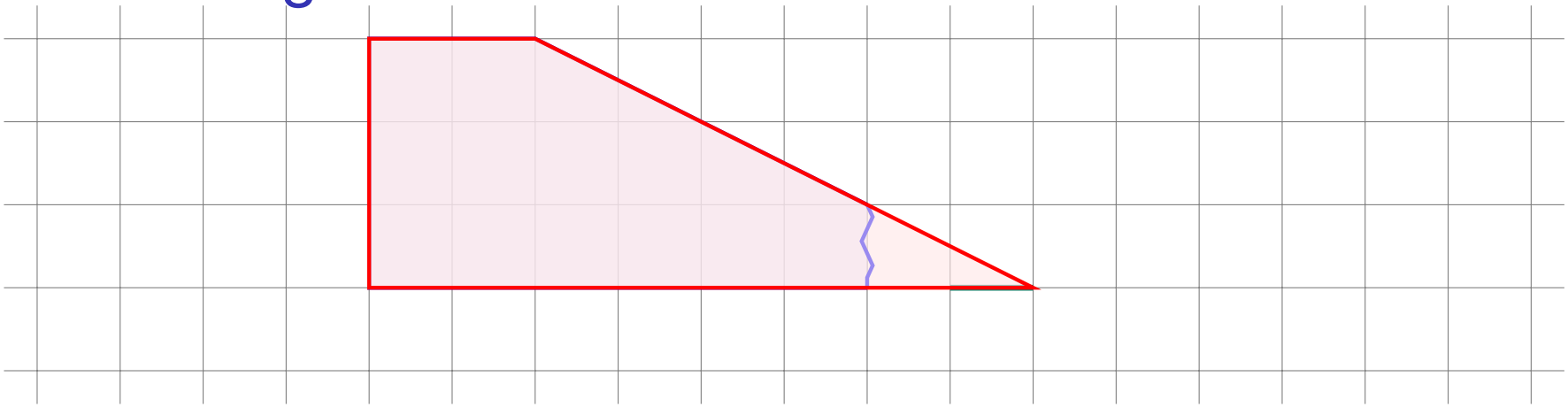

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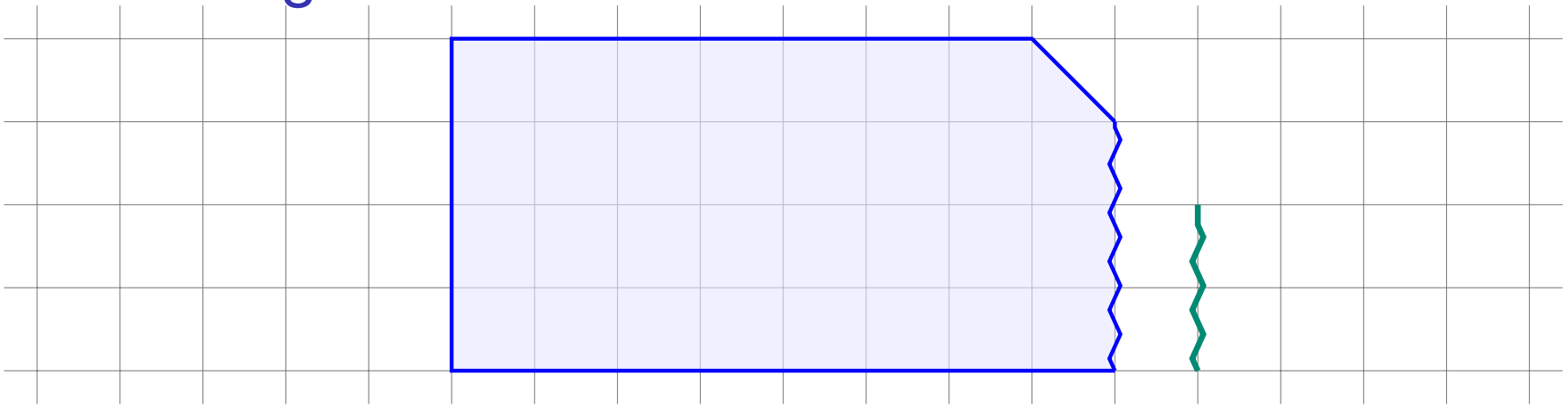

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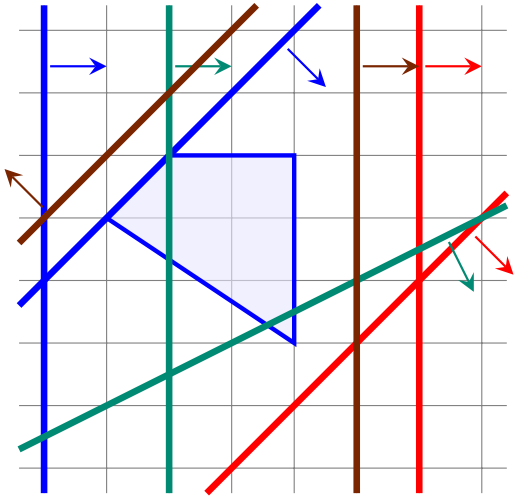

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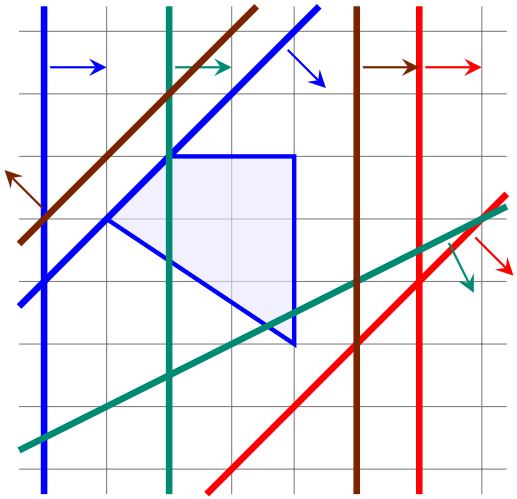

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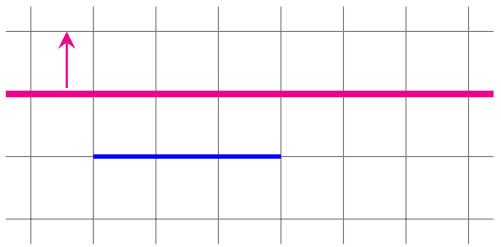

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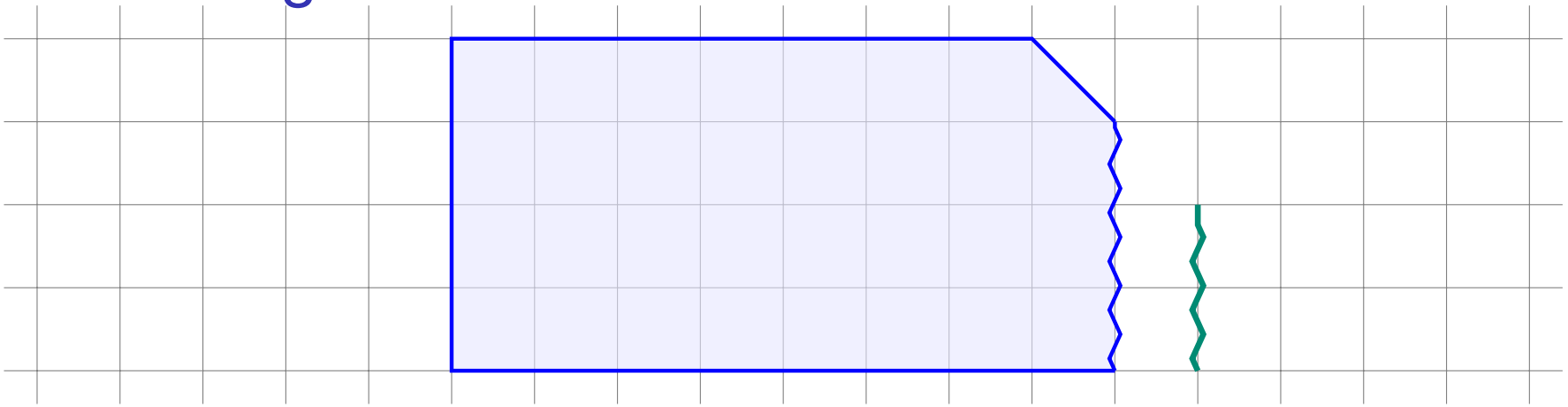

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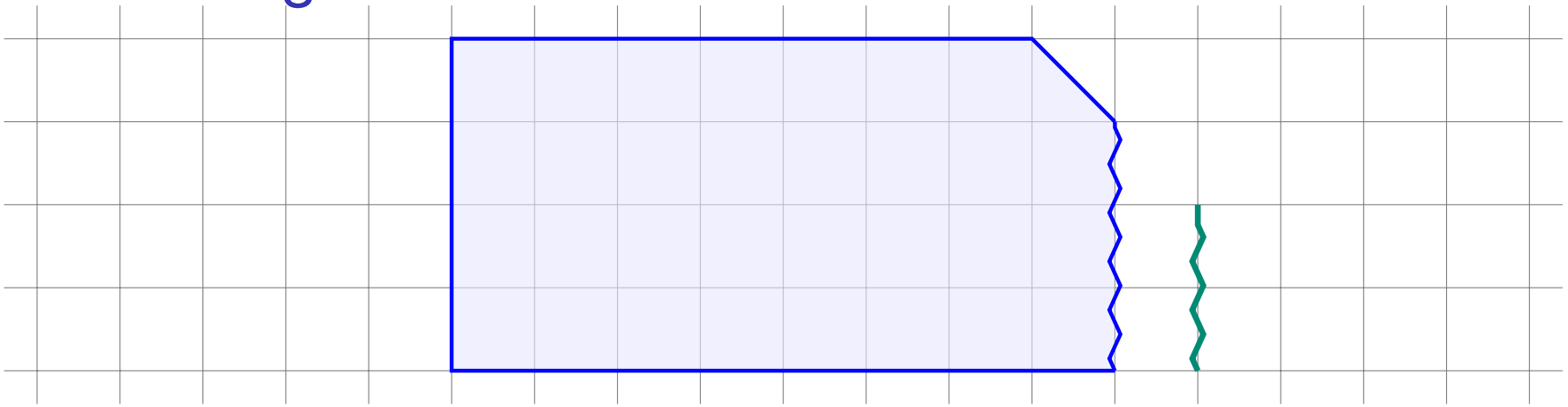

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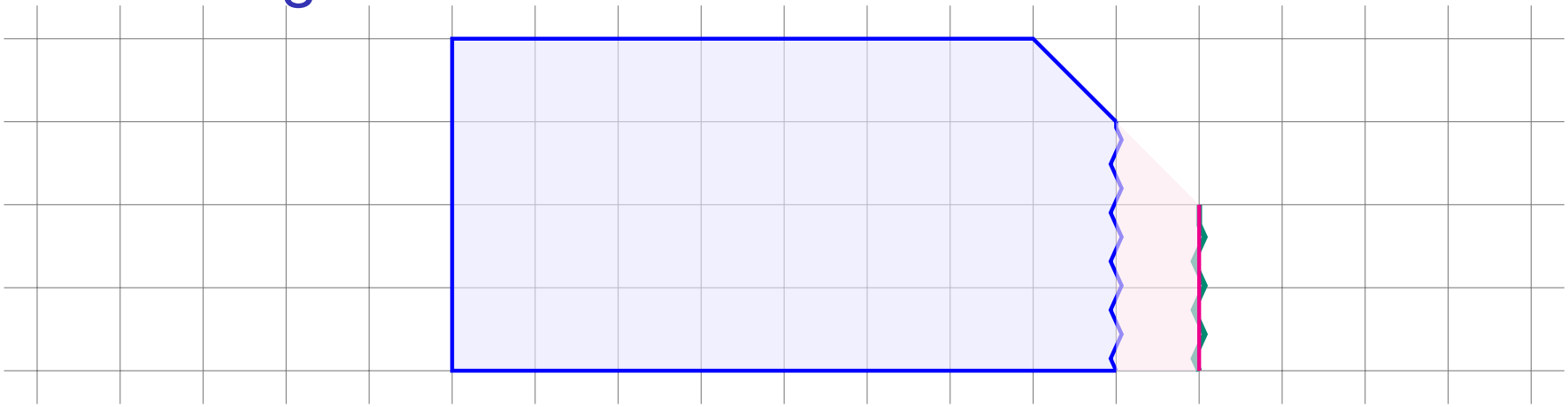

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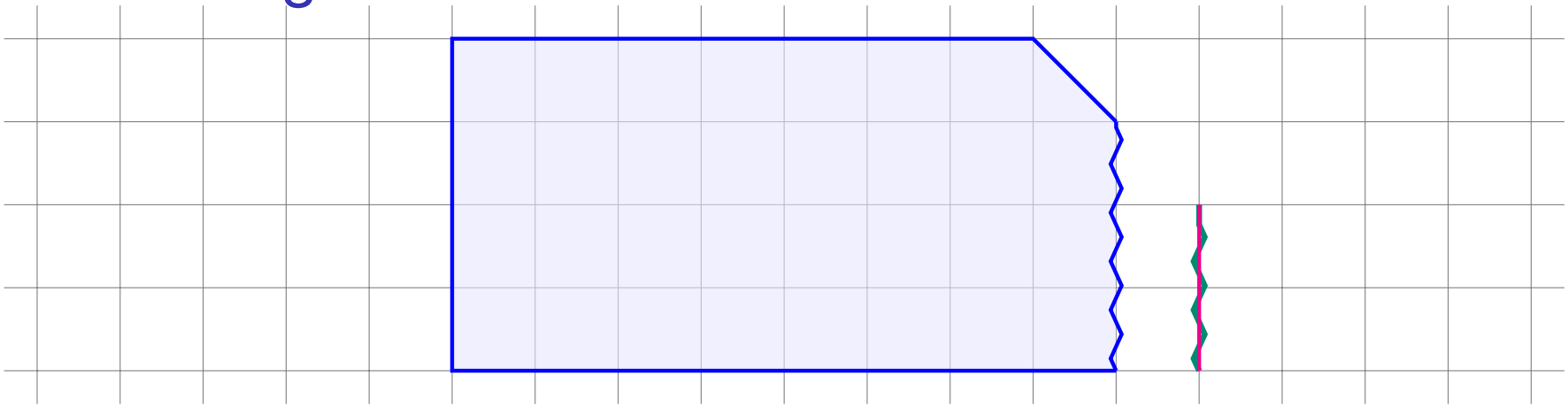

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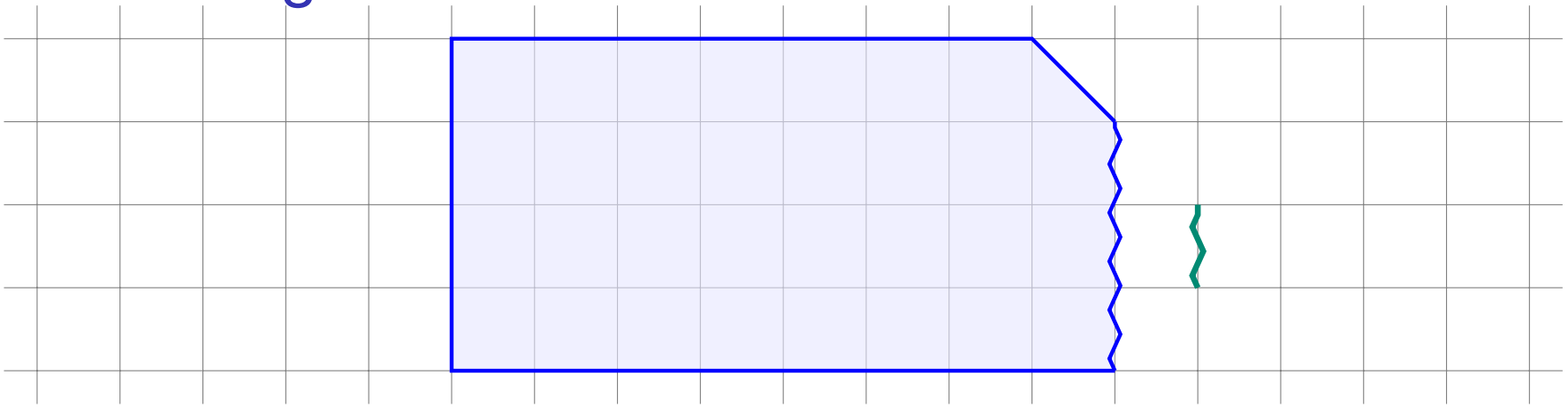

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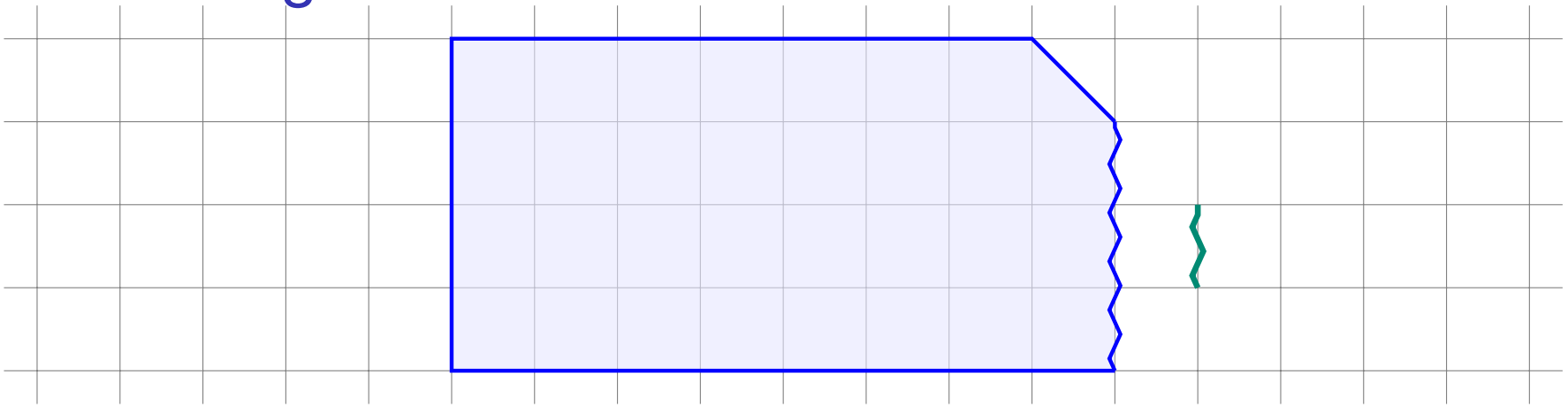

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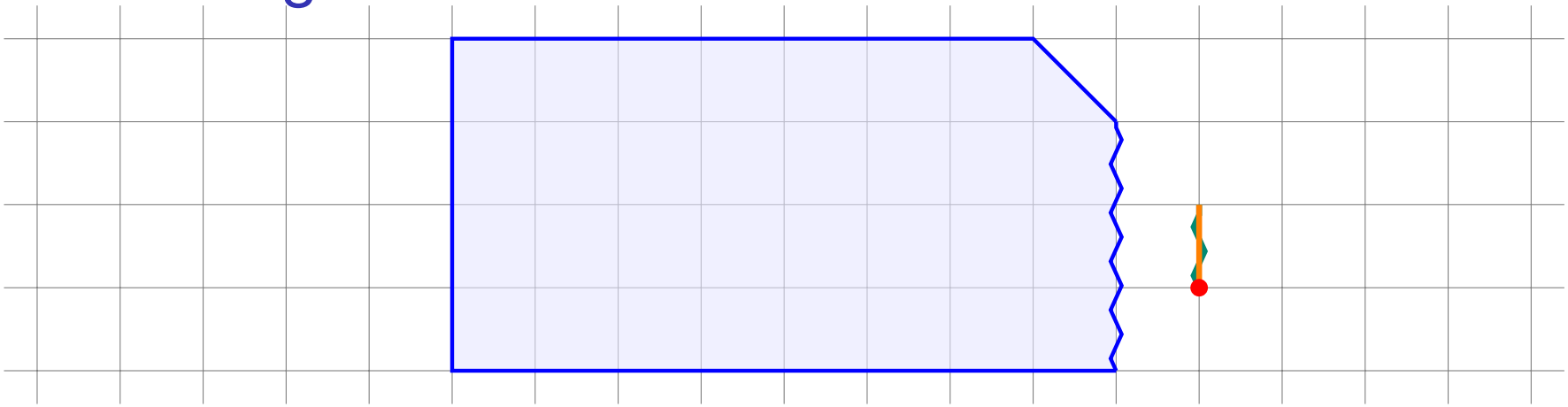

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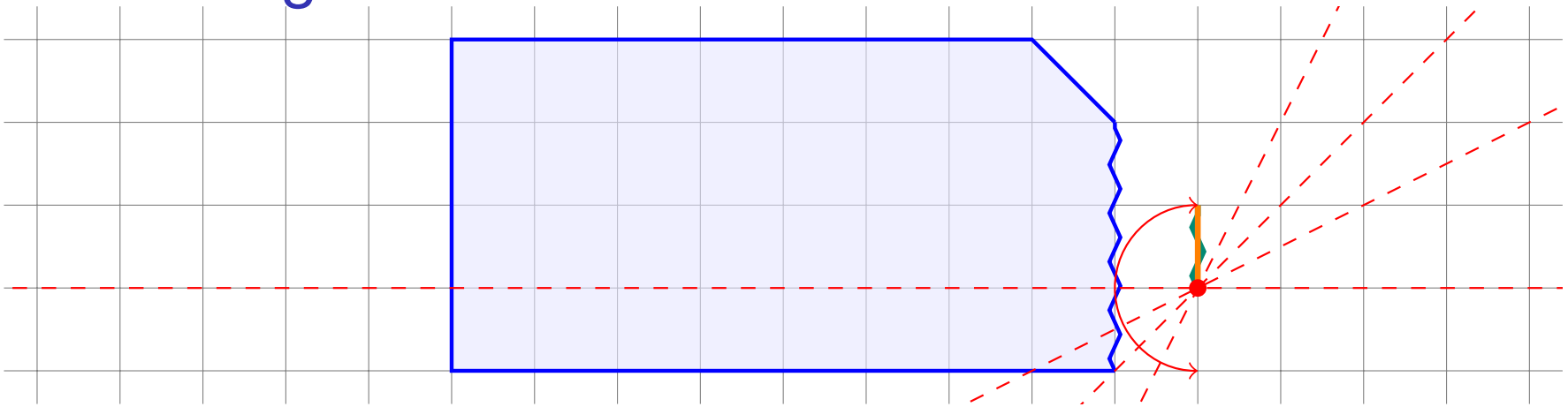

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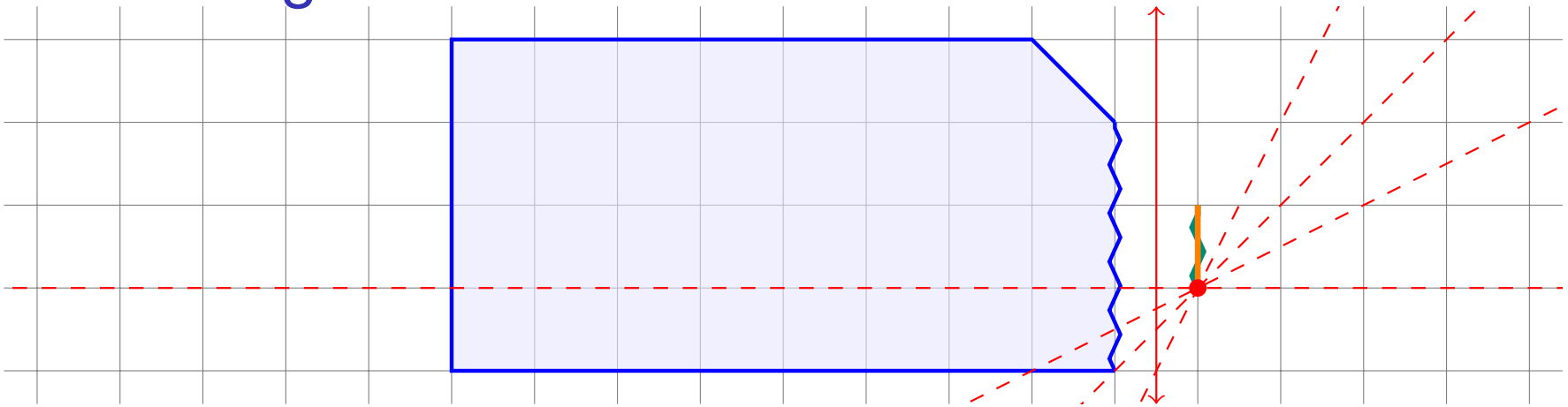

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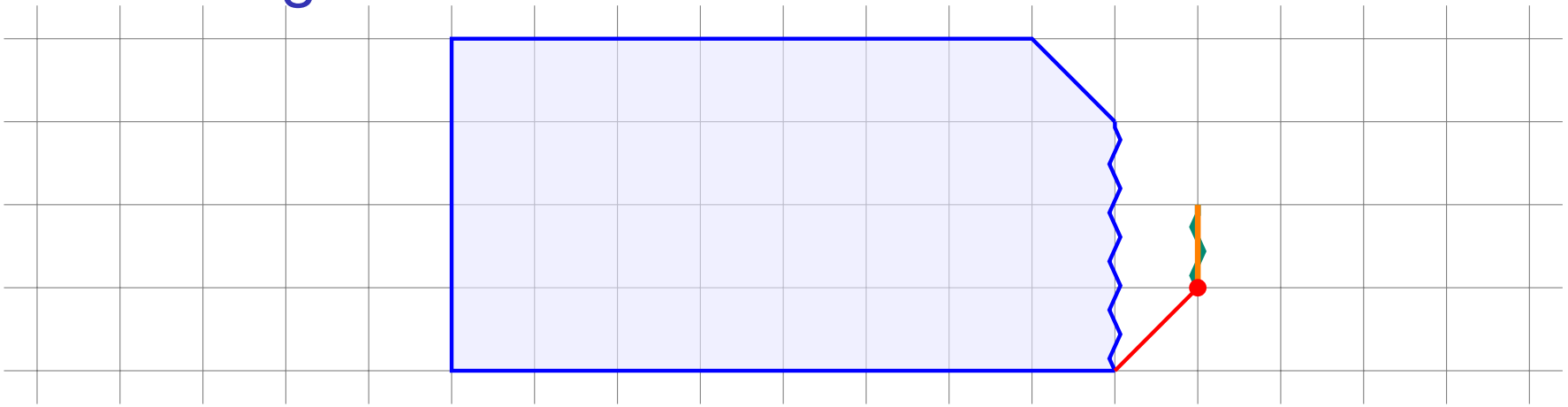

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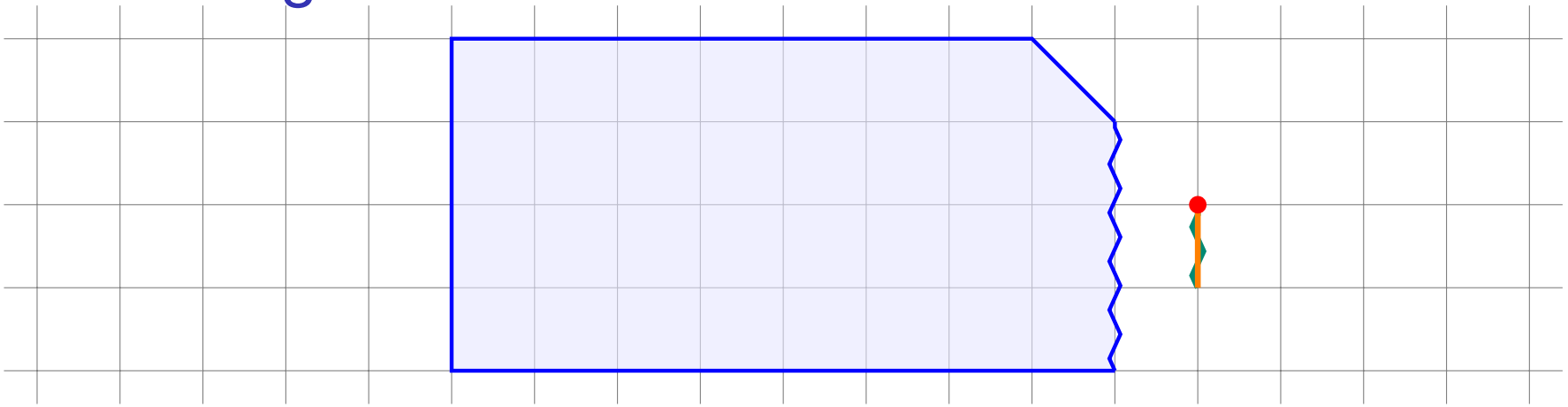

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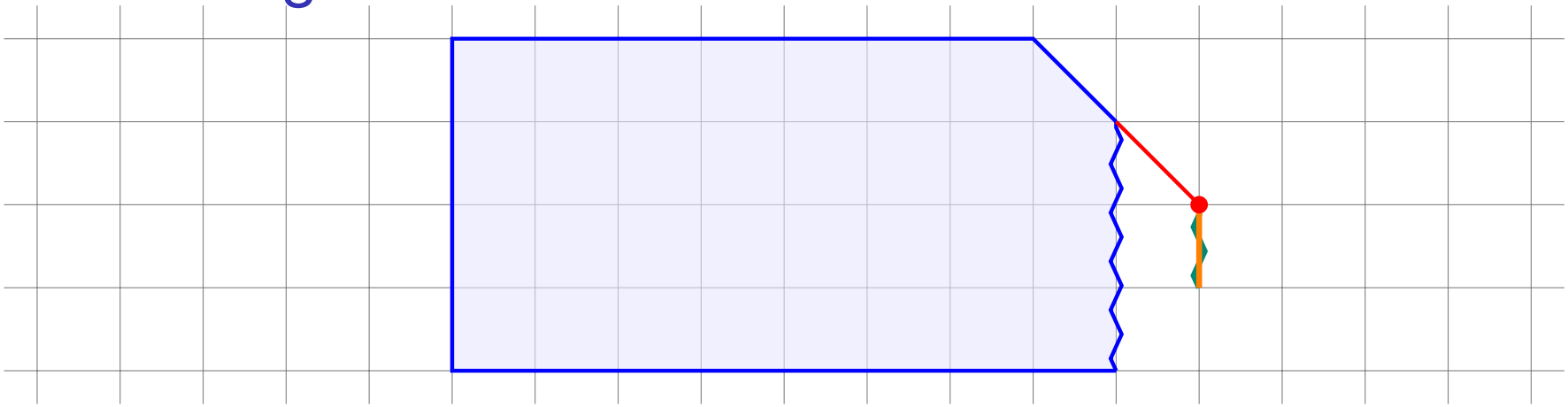

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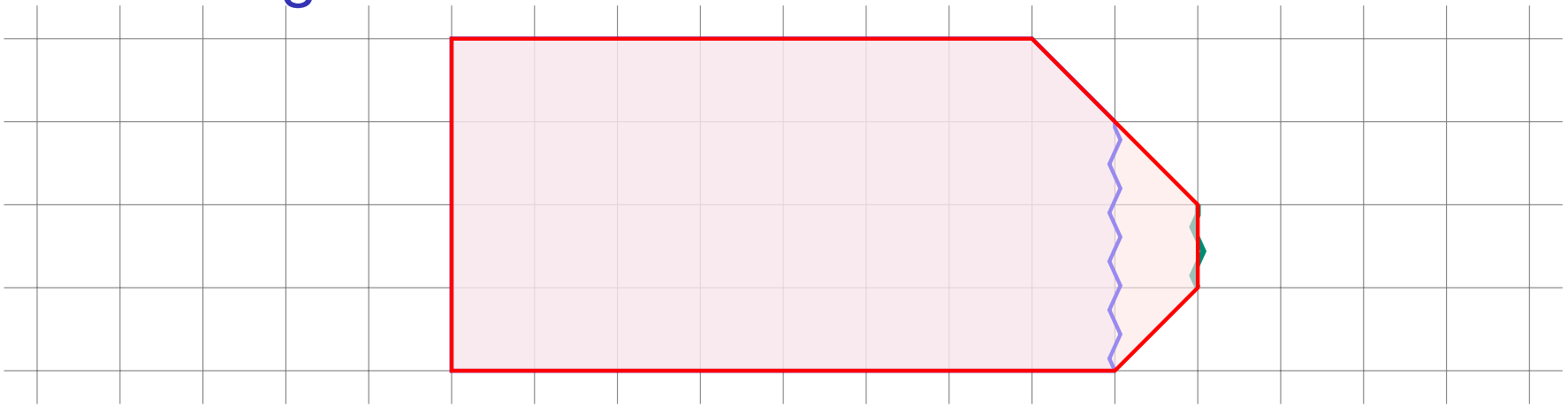

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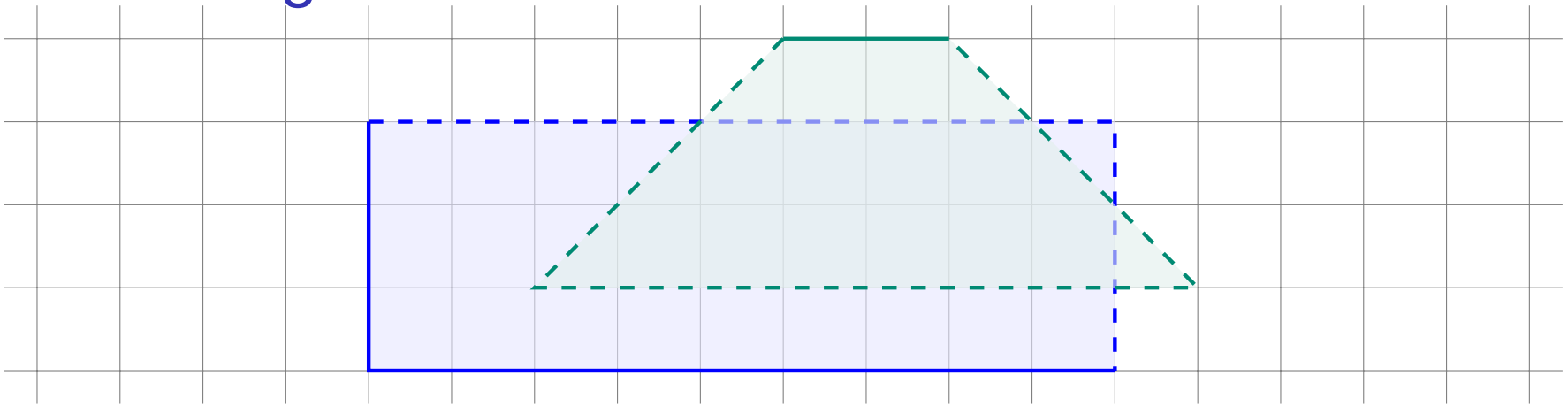

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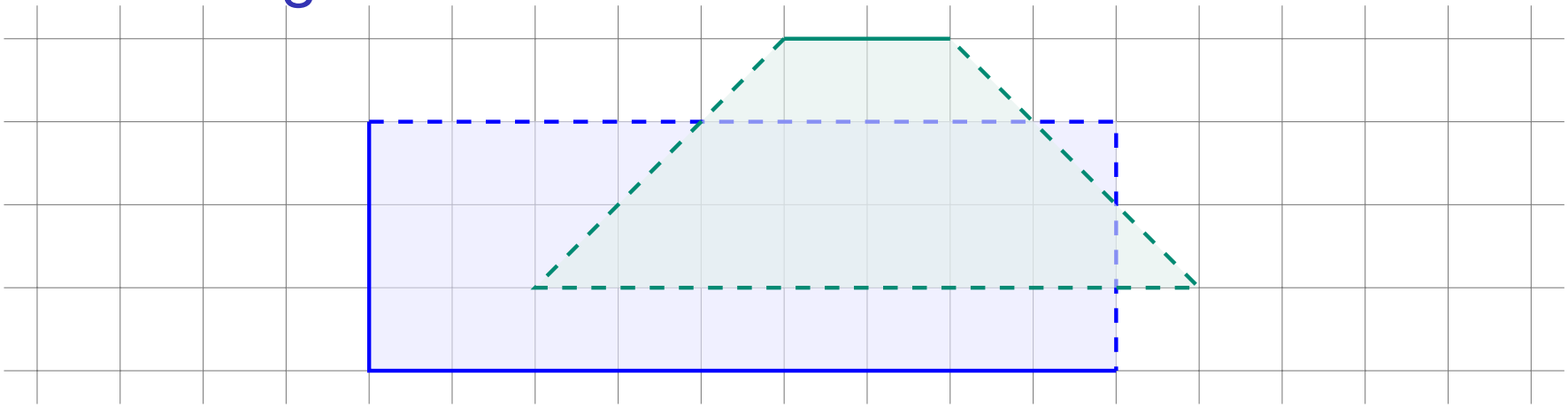

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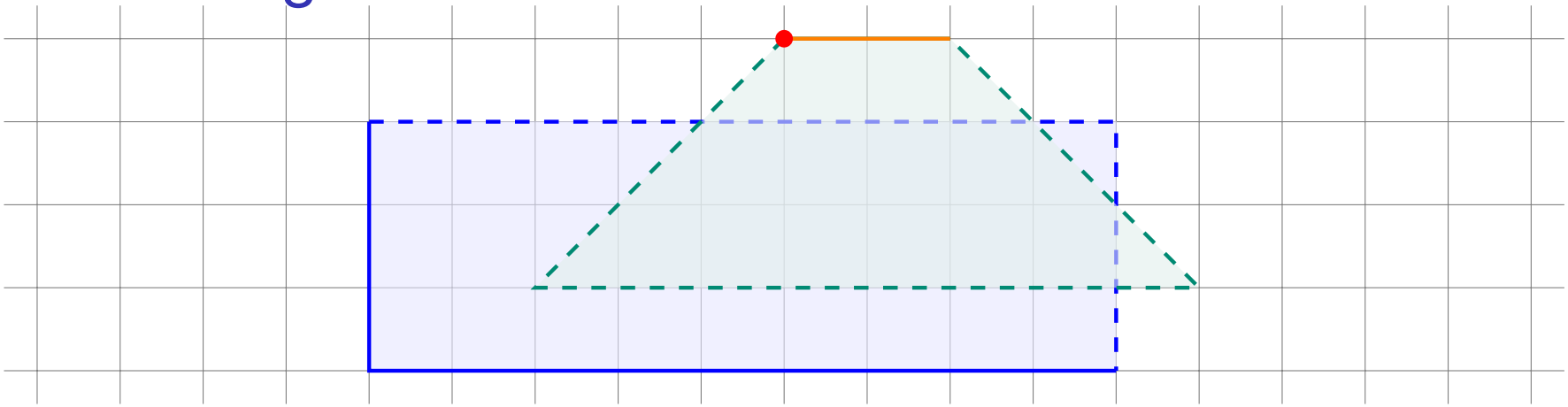

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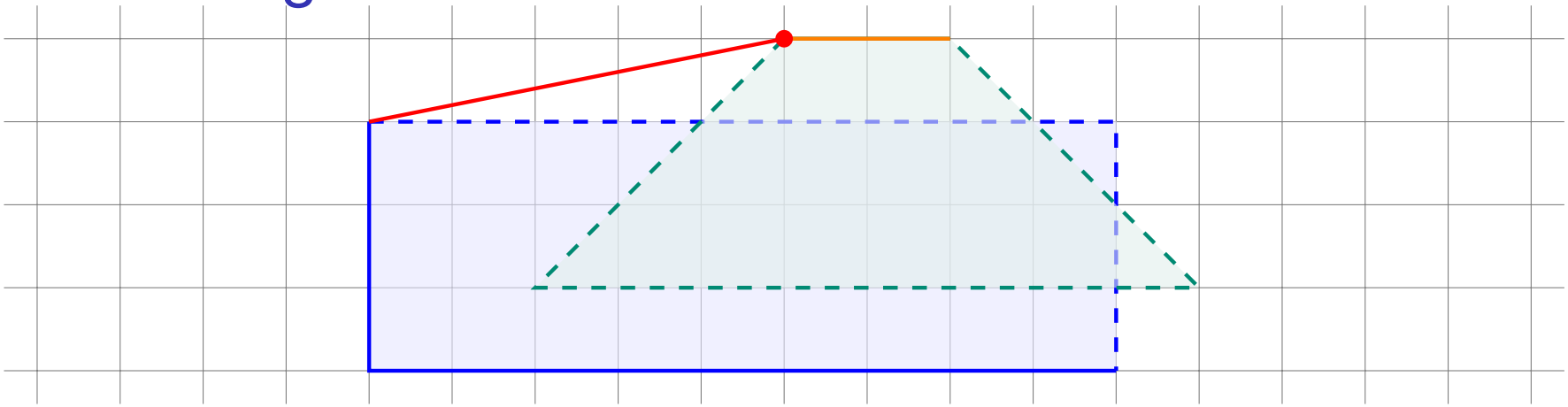

7. B extends beyond A by at most one and all cut constraints of B can be wrapped around shifted facet of A to include A

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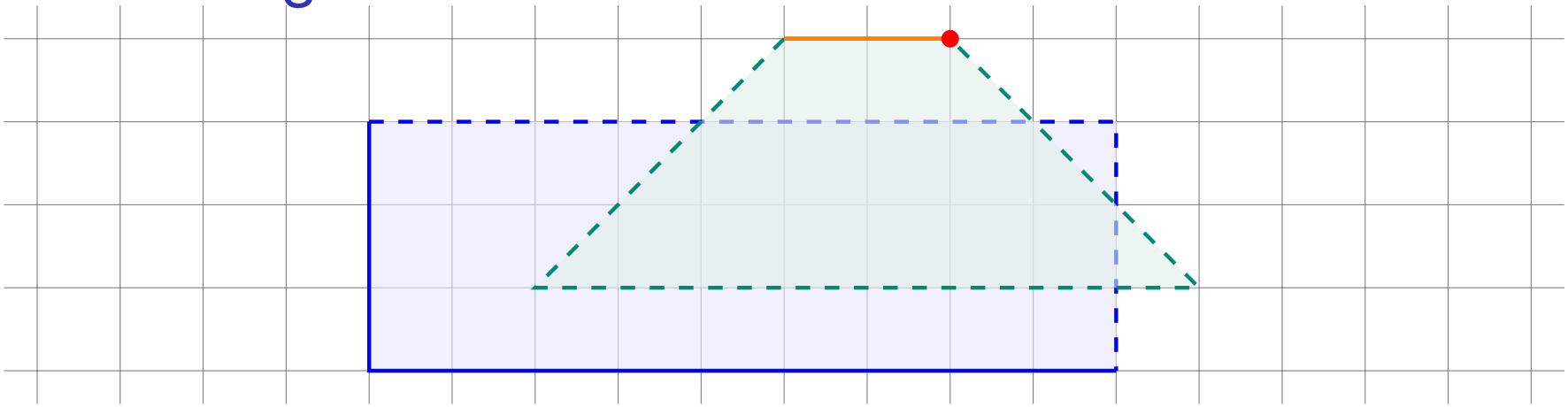

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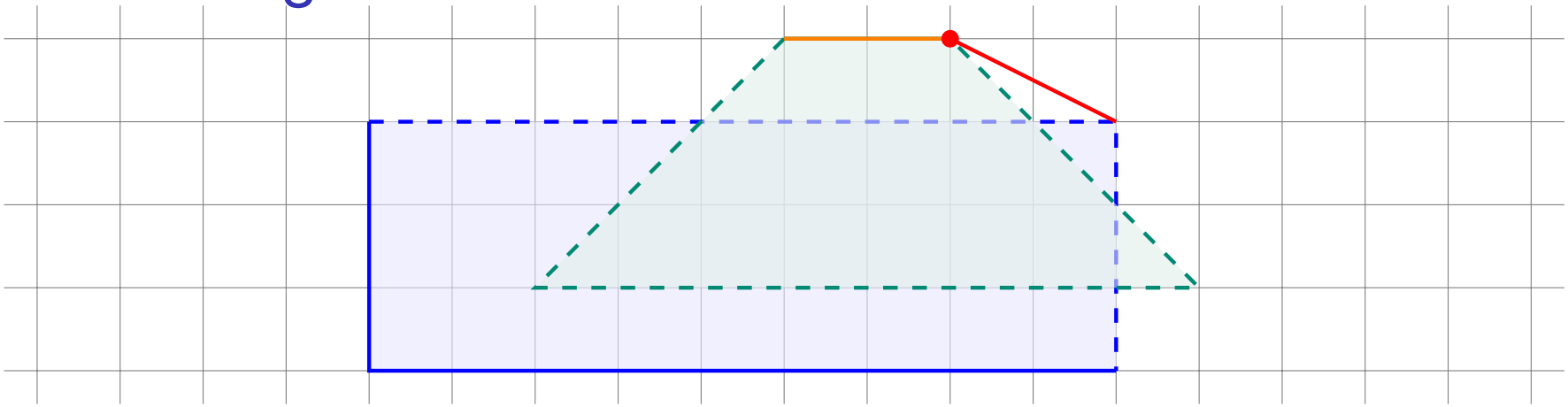

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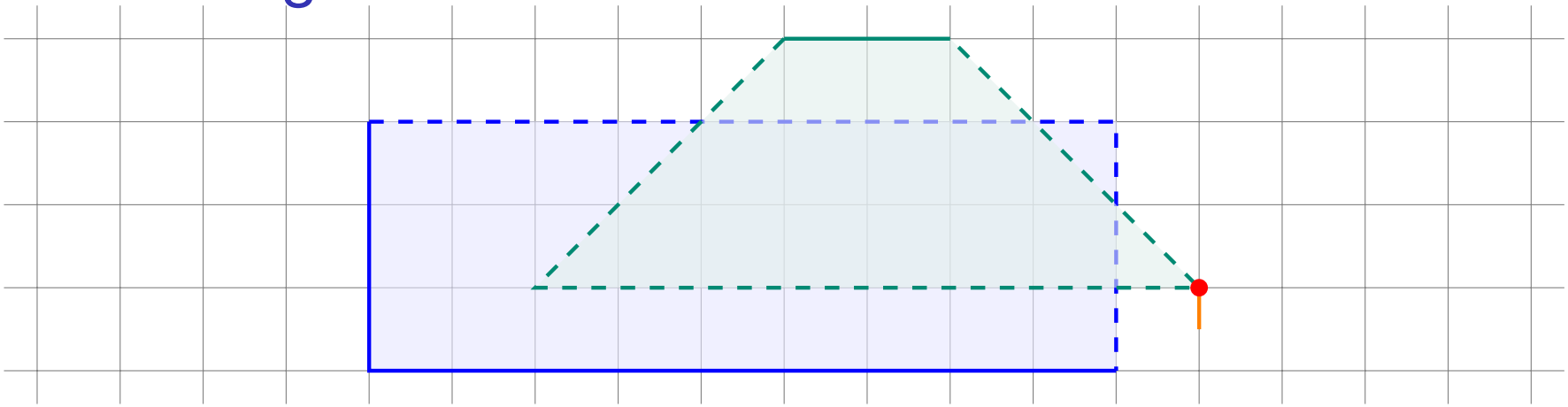

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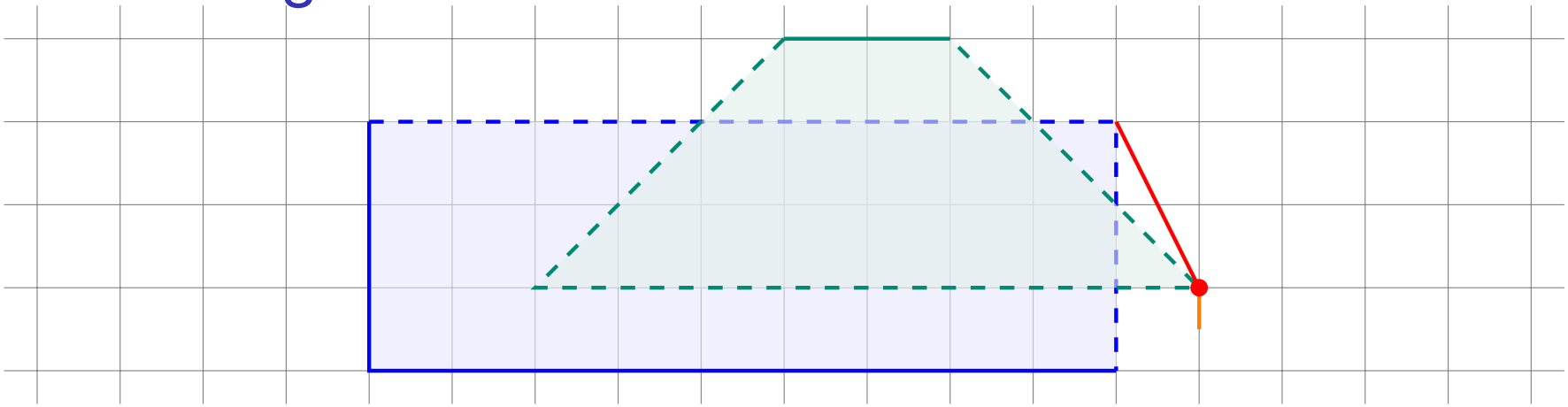

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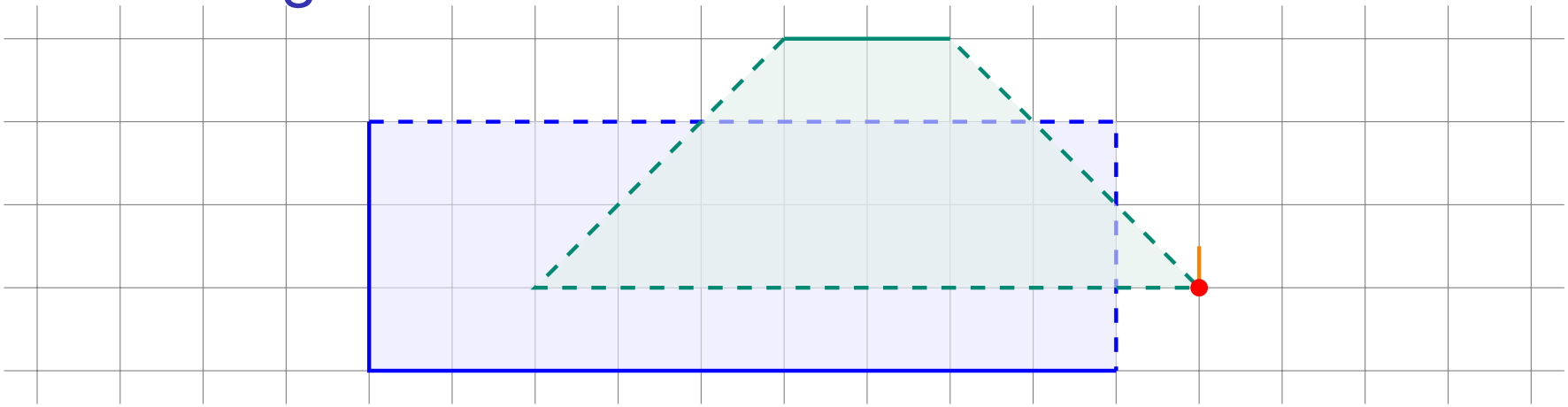

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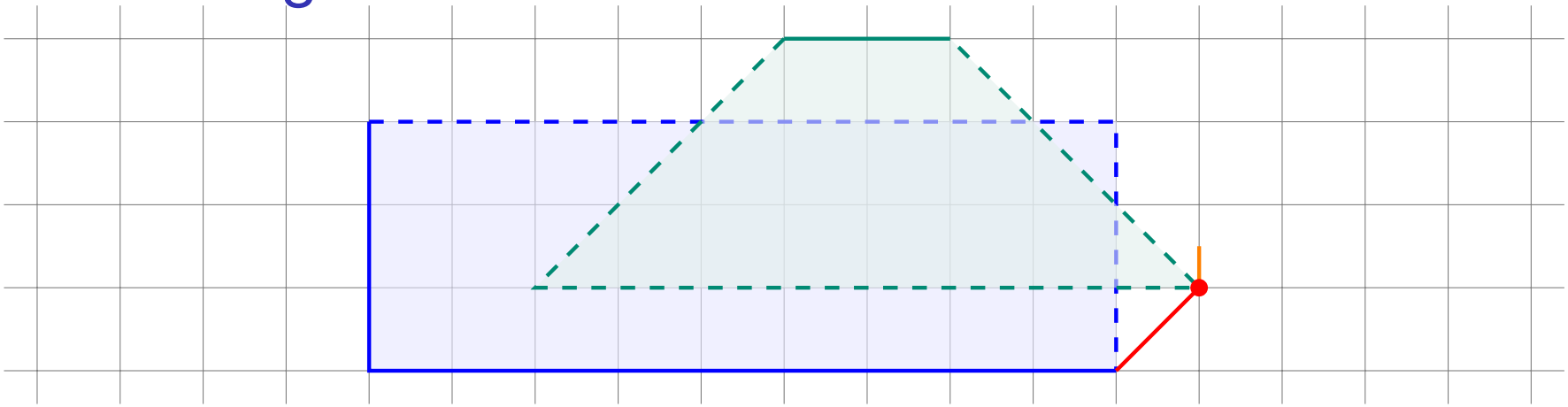

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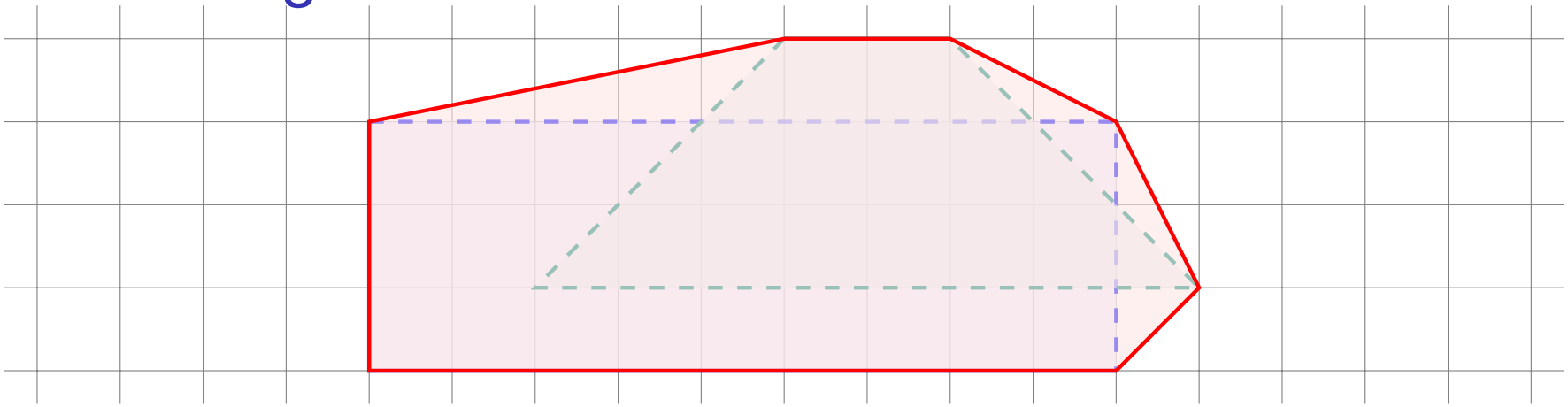

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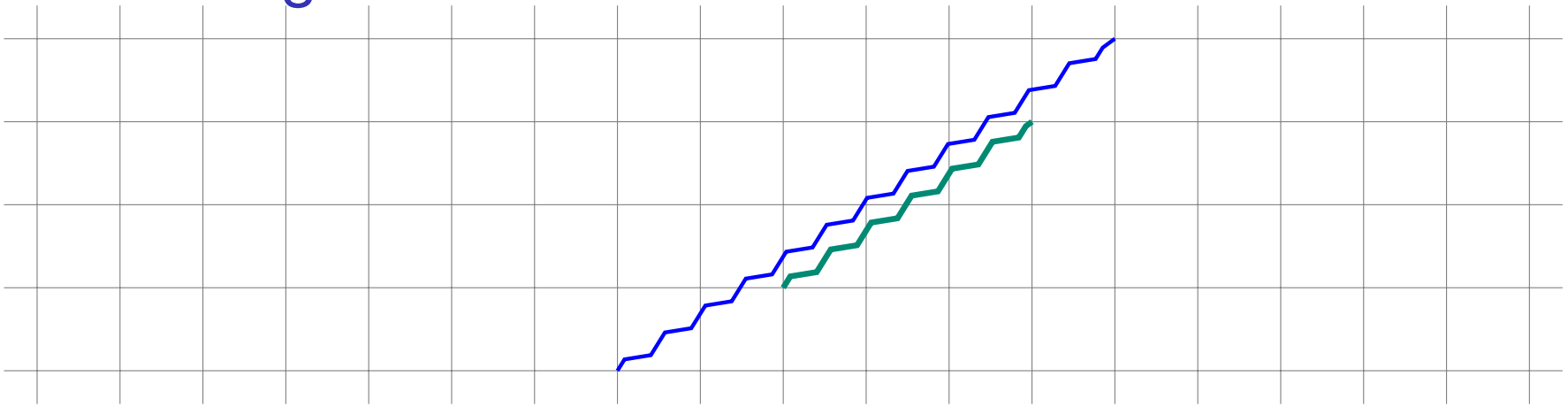

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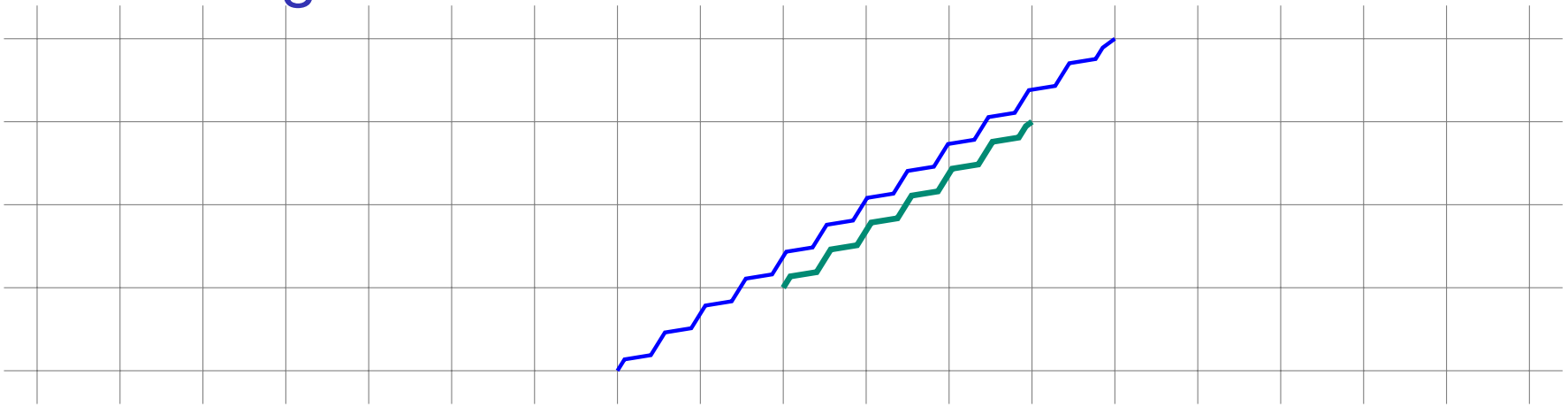

8. A has equality adjacent to equality of B
   Non-valid constraints of B (except $t(\mathbf{x}) \leq -1$) can be wrapped around $t(\mathbf{x}) \geq -1$ to include A
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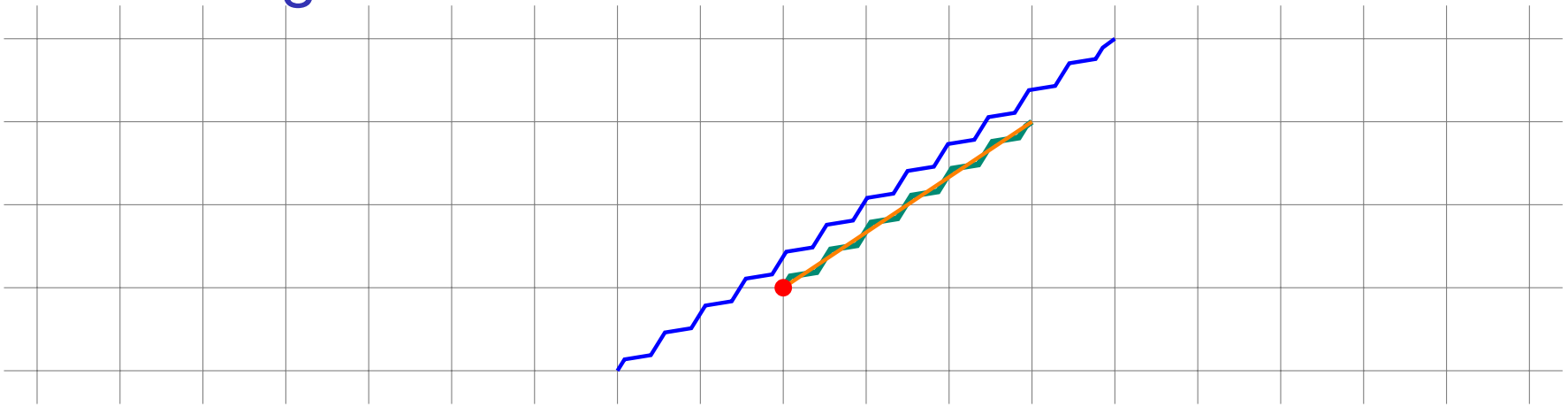

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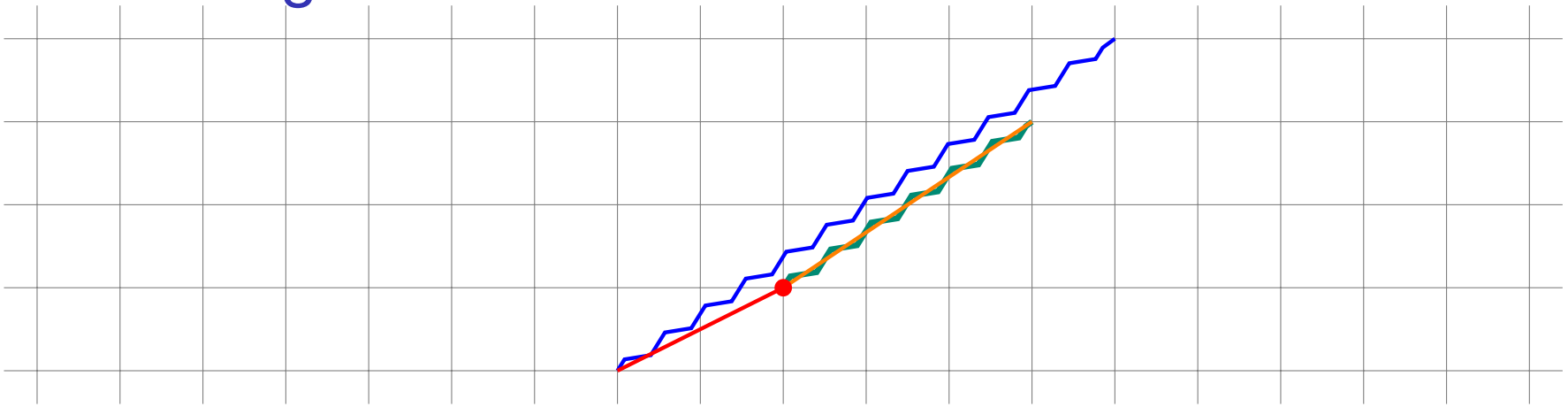

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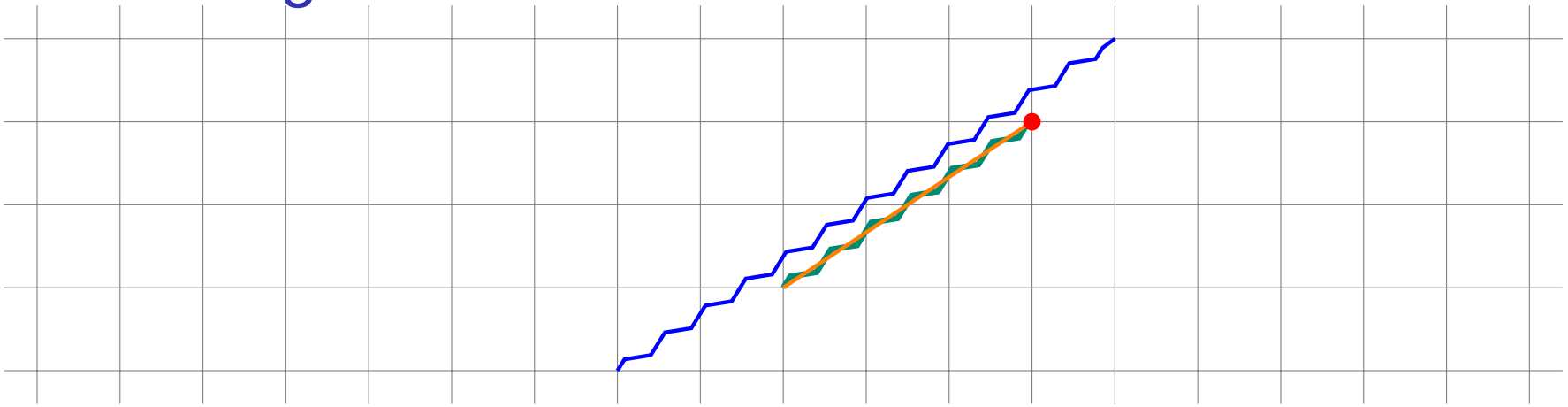

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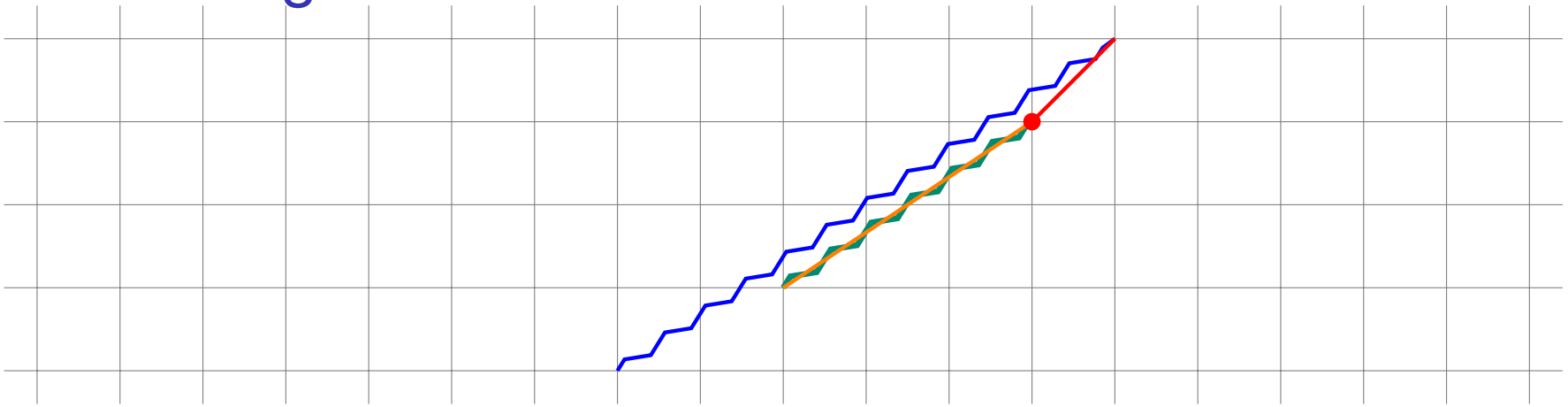

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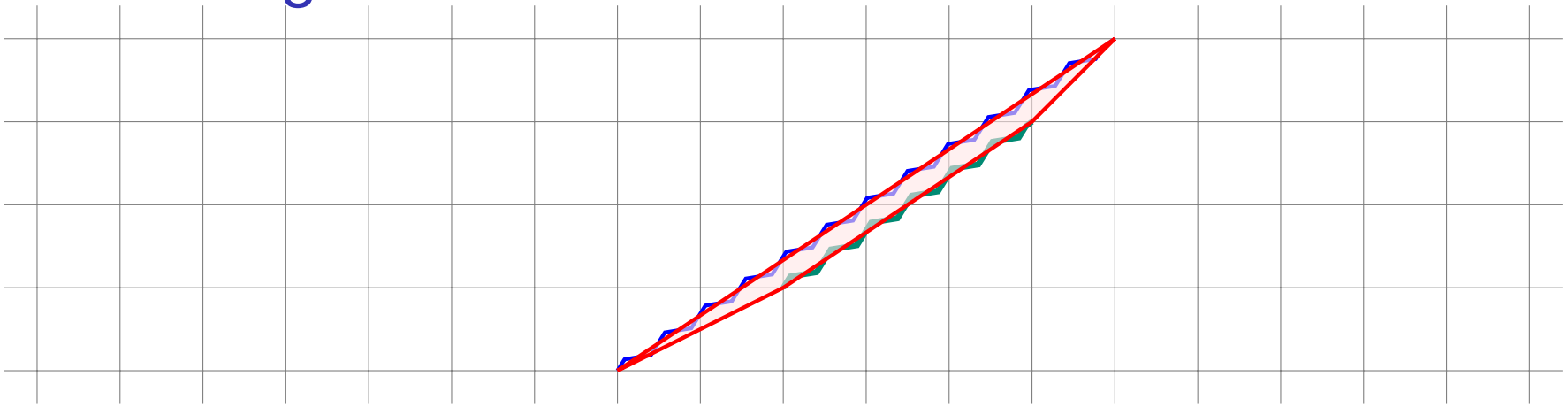

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Existentially Quantified Variables and Equalities

- Quantifier elimination in `isl` replaces existentially quantified variables by integer divisions of affine expressions in other variables
- These integer divisions are sorted prior to coalescing

- A and B have same number of integer divisions/existentials
  $\Rightarrow$ try all cases
- integer divisions of A form subset of those of B
  (after exploiting equalities of B)
  $\Rightarrow$ check if B is a subset of A
- integer divisions of B form subset of those of A and
  equalities of B simplify away the integer divisions of A not in B
  $\Rightarrow$ introduce integer divisions in B and try all cases

Outline

Conclusions

- it is important to keep the number of disjuncts in a set representation as low as (reasonably) possible
- coalescing in `isl`
  - never increases the total number of constraints
  - based on solving LP problems with same dimension as the original set
  - recognizes a set of patterns